

**025:250 COMPOSITION: ELECTRONIC MEDIA I**

Nov. 28, 2005

**Group Actions on Pitch-Class Sets**

1. Group actions on the octatonic scale  $S = \{C, C\#, Eb, E, F\#, G, A, Bb\} = \{0, 1, 3, 4, 6, 7, 9, 10\}$ .

Let the group  $H_1 = \{P, I_1\}$ .

Let the group  $H_2 = \{T_0, T_3, T_6, T_9\}$ .

Let the group  $G = H_1 \times H_2 = \{P, I_1\} \times \{T_0, T_3, T_6, T_9\} = \{P_0, P_3, P_6, P_9, I_1, I_4, I_7, I_{10}\}$ .

Let the group  $G$  act on the set  $S$  such that  $G(S)$  is represented by the table below:

|          | 0  | 1  | 3  | 4  | 6  | 7  | 9  | 10 |
|----------|----|----|----|----|----|----|----|----|
| $P_0$    | 0  | 1  | 3  | 4  | 6  | 7  | 9  | 10 |
| $P_3$    | 3  | 4  | 6  | 7  | 9  | 10 | 0  | 1  |
| $P_6$    | 6  | 7  | 9  | 10 | 0  | 1  | 3  | 4  |
| $P_9$    | 9  | 10 | 0  | 1  | 3  | 4  | 6  | 9  |
| $I_1$    | 1  | 0  | 10 | 9  | 7  | 6  | 4  | 3  |
| $I_4$    | 4  | 3  | 1  | 0  | 10 | 9  | 7  | 6  |
| $I_7$    | 7  | 6  | 4  | 3  | 1  | 0  | 10 | 9  |
| $I_{10}$ | 10 | 9  | 7  | 6  | 4  | 3  | 1  | 0  |

Notice that the actions of the group  $G$  fixes and interchanges 2-element subsets of  $S$ .

2. Group actions on Webern's row for *Concerto for Nine Instruments*, Op. 24, denoted as the set  $S = (G, B, Bb, Eb, D, F\#, E, F, C\#, C, G\#, A) = (0, 4, 3, 8, 7, 11, 9, 10, 6, 5, 1, 2)$ .

Let the group  $H_1 = \{P, I_5\}$ .

Let the group  $H_2 = \{P, R_6\}$ .

Let the group  $G = H_1 \times H_2 = \{P, I_5\} \times \{P, R_6\} = \{P_0, I_5, R_6, RI_{11}\}$ .

Let the group  $G$  act on the set  $S$  such that  $G(S)$  is represented by the table below:

|           | 0 | 4  | 3  | 8 | 7  | 11 | 9 | 10 | 6  | 5 | 1  | 2  |
|-----------|---|----|----|---|----|----|---|----|----|---|----|----|
| $P_0$     | 0 | 4  | 3  | 8 | 7  | 11 | 9 | 10 | 6  | 5 | 1  | 2  |
| $R_6$     | 8 | 7  | 11 | 0 | 4  | 3  | 5 | 1  | 2  | 9 | 10 | 6  |
| $RI_{11}$ | 9 | 10 | 6  | 5 | 1  | 2  | 0 | 4  | 3  | 8 | 7  | 11 |
| $I_5$     | 5 | 1  | 2  | 9 | 10 | 6  | 8 | 7  | 11 | 0 | 4  | 3  |

Notice that the actions of the group  $G$  interchanges 3-element subsets of  $S$ .