The Design of a System for the Generation of Arbitrary Periodic Waveforms in the 40 cps to 4 kc Range

by

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ABSTRACT

The Arbitrary Waveform Generator (AWG), an instrument intended for the electronic production of musical sounds, has been developed. The waveforms are generated by electronically plotting a curve and periodically presenting it at the AWG output. The time scale of the waveform to be produced is divided into 96 equal time elements and 96 values of the function are set into the AWG memory (by the operator) in the form of dc levels. These 96 values of wave amplitude are commutated repeatedly, each appearing at the output for the duration of one time element. The resulting waveform is filtered to remove discontinuities. With the filters used in the model built by the author, it is shown that the 16th harmonic is resolved with negligible error. The rate at which the given waveform is repeated is continuously variable from 40 cps to 4 kcs and is controlled by a square wave generator. Examples of the AWG waveforms are given in the form of scope trace photographs showing both unfiltered and filtered forms of the output. A magnetic tape recording which accompanies this thesis gives further examples of the resulting tones.

A system is proposed by which the waveform repetition rate could be derived from an external sound source rather than the square wave generator. The fundamental frequency of the external source is multiplied by 96 and used to drive the commutator, producing a waveform with a repetition rate equal to the frequency of the controlling source. In this way, the AWG would function as a waveform converter. The system described would make the frequency multiplication with an accuracy of 0.1% and with a response time of two cycles at the input frequency. This would allow the generated waveform to follow a pitch sequence of the sound source as closely as could be resolved by the ear.

A description of computer programs used for the generation of charts for use in loading the AWG memory is provided. Actual charts are included which give the values of sin (nx) evaluated at 96 equal values of x over 2π.

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I. INTRODUCTION

The piece of equipment to be described herein was conceived to fill a need in electronic music. This branch of musical composition, still in its infancy, does not yet begin to exploit fully the potential of electronics in its present state of art. It is hoped that the device realized through this research is at least a small step in that direction. Inasmuch as some of the readers of this paper will not be well acquainted with electronic music, a brief discussion of its goals and techniques follows.

The techniques of electronic music may be put into perhaps three or four classifications, but in any studio, one may find two or more of these combined. The first of these is musique concrète, wherein recordings of "real" sounds of all kinds (for example, musical instruments, voices, or perhaps ever street noises) are electronically modified into new sounds which are then organized into a musical composition. The second classification emerged from a desire for greater control over the sound sources and has substituted square wave generators, sine wave generators, random noise sources, and the like, for the real sounds.

The modification techniques however remain much the same. The most recently evolved is that of computer music which is perhaps really of two types. One of these types uses a digital computer to synthesize, by means of an inverse time sampling technique, waveforms which are realized at the output by a digital to analogue converter. The other use of a computer which is in a less strict sense electronic music, is that in which the computer is actually used to choose which notes are to be written (within certain limits given it by the composer) [Hiller, 1958]. Inasmuch as computer time is expensive and computers are much too general for efficient production of music, widespread use of these techniques have not occurred.

Electronic generation and modifications of sounds, as described above, is found in almost all electronic studios, which number perhaps a few dozen in the United States today. However, since the only additional equipment needed for musique concrète is a microphone and perhaps a portable tape recorder, the two are very frequently combined. A list of sound sources generally found in the conventional electronic music studio is given in Table I below.

TABLE I

Representative Sound Sources

Generator	Spectrum
Sine wave	fundamental only
Square wave	all odd harmonics--amplitude decreasing as $1/n$
Saw tooth	all harmonics--amplitude decreasing as $1/n$
Triangular wave	all odd harmonics--amplitude decreasing as $1/n^2$
White noise source	all frequencies in audio spectrum with flat power density
Pulse generator	all harmonics in audio spectrum --amplitudes essentially equal

Any or a combination of these sources may be used directly as a sound or may be modified by devices such as are found in Table II.

TABLE II

Representative Modifiers

Device	Result
Linear mixer	adds amplitudes of inputs
Band pass filter	selects specified band of the spectral components of a sound
Band elimination filter	supress specified band in the spectrum of a sound
Balanced or product modulator	produces the product of two waveforms
Amplitude modulators	one waveform becomes the envelope of another

Other equipment consists of tape recorders, amplifiers, gating circuitry, monitoring facilities, timing devices, and splicing and editing equipment, to name the more important items. No two studios will be alike in their specific equipment make-up but they will contain most of the basic items listed above in addition to other locally developed equipment.

The goals of the composer of electronic music are difficult to capsulize. Perhaps we might start by clearing up a common misconception. He is not interested in electronically synthesizing the sound of an orchestra performing in Carnegie Hall because an orchestra performing in Carnegie Hall can do this best! What he does want, rather, is to produce what conventional musical instruments and musicians cannot produce. This would include intricate rhythm patterns that performers would find difficult or impossible to execute, production of sounds which have spectra different from that of any conventional musical instrument, and, further, to manipulate tones in ways that a conventional instrument cannot functionally operate. In short, the composer wishes to be able to create any sound sequence he can imagine (or perhaps a few he cannot imagine!)

and to be able to organize them into a composition. By "organize" here, we mean not only an ordering of musical forms in his mind or on paper but also the synchronizing of these sounds on tape so that his composition is a reality. Unfortunately, most studios fall short of these goals.

There are two basic methods of obtaining a desired sound with the equipment outlined above, additive synthesis and subtractive synthesis. Mixing together the various components of a sound is additive synthesis. Unless one has many generators of the type mentioned above, rerecording plus mixing is required --with the inherent loss of signal-to-noise ratio with each recording. In subtractive synthesis, one generally starts with the white noise source or some other complex sound and removes with a filter unwanted spectral components. This process is also quite limited by the bandwidths of the filter that the studio is likely to possess. (Commonly available tunable filters which have a bandpass at no less than 1/3 octave can, at best, transform white noise into "colored" noise.) Thus, unless the studio includes "special purpose" equipement in addition to the above, the composer will find himself unable to produce many of the sounds he would like.

Once the sounds are produced, the composer must face the problems of organizing and synchronizing them on tape. Eventually, the composer must find himself very laborously cutting tape, splicing, dubbing, editing, mixing, building his music inch by inch on tape, many times working weeks or months for a few minutes of music. Unfortunately the composer is often forced into the unhappy compromise of "wanting what he produces rather than producing what he wants".

Many have been searching for solutions to these difficulties, trying to find ways to control more parameters of a sound with less effort or even to produce "real time music". Paramount among these attempts is the RCA synthesizer at the Columbia-Princeton studios [Olson and Belar, 1955] (probably the best equipped studios in America). The RCA Synthesizer is a binary paper-tape controlled bank of equal-tempered pitch oscillators which may be controlled in many parameters so as to obtain four simultaneous instrumental voices. No other studio has been able to afford such an elaborate piece of equipment, yet the RCA Synthesizer need not be considered the ultimate. Although there are other attempts, too numerous to mention, at such automation on a lesser scale, one more will be cited here for the purpose of comparison with the piece of equipment to be described subsequently.

A harmonic tone generator has been designed and built at the University of Illinois studios by James Beauchamp [1963, 1964]. His device produces a tone containing six harmonics, independently variable in amplitude. Phase control of the second harmonic has also been incorporated. These, as well as the resulting waveform periods, are voltage controlled from some external source. This device will be more thoroughly discussed in Chapter III.

II. PERCEPTUAL PARAMETERS OF MUSICAL TONES

There are three characteristics of sound waves that are of interest in the physical sense: frequency, intensity, and wave shape. By controlling these three parameters, one could create any sound wave possible. However, the time variation of these three quantities is quite complex and we find that at the perceptual level, there are many more than three parameters. Although the term "perceptual parameter" seems to suggest a subjective nature (which is indeed the case), it is still true that the time variation of the three physical parameters mentioned will completely specify the sound experience for a given audience. In electronically generating a musical tone, the problem is not in the method of controlling these parameters but rather in the knowledge of the function specifying the variations.

Pitch is predominantly dependent upon frequency, but is also a function of intensity and waveform. (However, for a complex waveform, pitch is not a well defined term.) A complete discussion of this dependence can be found in any text on acoustics [e.g., Beranek, 1954] and would serve no real purpose here.

We will just note in passing that many aspects of the perception of music are "learned", as for example, an unbiased auditor will not identify an octave in frequency as a doubling in pitch. The ear is quite sensitive to change in frequency and is increasingly so at higher frequencies. There is slight disagreement in the literature as to what the minimum perceptual change in frequency is, but it can be bracketed between 0.1% and 0.5%. Our own experiments seemed to show that the 0.1% figure is correct but it is difficult to tell whether or not the auditor was completely unbiased.

Loudness is chiefly dependent upon intensity, but is also a weak function of frequency. Extensive measurements of this dependence have been made by Fletcher and Munsen [1933] for pure tones and by Pollack for noise-bands.

Timbre has an even more complex dependence on the physical parameters than do loudness and pitch. One usually considers timbre as dependent upon harmonic structure and modified by frequency and intensity. Olson [1942] suggests that timbre is best described by "the instantaneous cross section of the tone quality". The fact that we could find no better definition of timbre than the above, testifies for its subjective nature. An often quoted example of one aspect of this subjectiveness (and was easily verified in our own studio) is that of the "missing fundamental". If pure tones of frequencies 400, 600, and 800 cps are linearly added (so as not to produce any product terms) the ear will perceive this as being a waveform with a fundamental of 200 cps. Furthermore, if tones of 500 and 700 cps are added to the above combination the resulting sound is perceived to change to a 100 cps fundamental. It is believed that the ear is a nonlinear device and thus the "missing fundamental" is supplied by the product terms therein.

Two other variations may be superimposed upon the above parameters: vibrato and tremolo. Tremolo is an amplitude modulation of the waveform while vibrato is a frequency modulation of the waveform. Although these are largely under the control of the instrumentalist (or vocalist), they are usually small variations of about 7 cps. Due to the fact that these two variations are perceived as very nearly the same effect, and that in most instruments the two effects occur in combination, there is some confusion in the literature of these two terms. Physically, however, they are quite well defined.

The envelope of the waveform for a musical tone is generally divided into three temporal elements: the attack-transient, the steady state, and the decay-transient. The attack is usually 0.1 sec or less in duration while the decay is usually longer, perhaps even up to a few seconds. Certainly, for electronically produced music, though, there are no restrictions on these lengths of time. The attack and decay durations are quite important to the result which is perceived as is evidenced by the fact that a piano recording, played backwards, sounds very much like a pipe organ. Of the three, the steady state duration is least important perceptually and only determines the length of the note.

If what we have considered so far were the whole story, the electronic generation of music could be perhaps rather straightforward, but such is not the case. It has been shown quite definitely that there exists a microscopic detail of musical sounds which is perceptually equally as important as the above. The exact nature of these variations is not known to any great extent but Clark and Miller [1964], Clark et al. [1963], and Luce [1963] have done much work in this area. Using a digital computer, Luce has conducted a time dependent spectral analysis of 14 nonpercussive instruments, concentrating mainly on regularities existing in the attack-transient and the steady state. Luce has found that on the microscopic level, there are both amplitude and waveform modulations which are important in determining the timbre of a sound. He has used these terms in the following sense: Amplitude modulations are those variations which do not affect the relative amplitudes of the spectral components whereas waveform modulations do. In addition, there appear to be random variations in phase and amplitude of each component which Luce believes to be important in making a sound "live".

Luce points out that the ear of necessity integrates over a short time in the perception of a sound, and a listener is capable of tracking temporal changes in harmonic structure subject to the constraints imposed by a time constant of perhaps 0.2 sec.

One may easily conclude that in order to completely specify an arbitrary musical tone, very high information rates are required. Of course, for the generation of a tone electronically, any information rate less than this right down to that of specifying an unmodulated sine wave is still sufficient for a note to be played. A sine wave, however, is quite uninteresting. In fact, a simple unmodulated combination of the Fourier terms from the steady state of some musical instrument usually sounds rather lifeless. Much more work is needed to determine just how much variation must be specified in order to produce interesting and lifelike sounds.

III. GENERAL DESCRIPTION OF THE AWG

No known generator has the capability of control of all the significant parameters of music. Such is financially and technically impossible at this time. This research, then, was aimed at a somewhat lower but more realistic goal, that of producing an instrument which can control only a few of these parameters, but in a new way. The Arbitrary Waveform Generator (AWG) is a significant piece of equipment, then, for two reasons:

(1) It can synthesize <u>any</u> periodic waveform with a very good accuracy.

(2) Circuitry is included which causes the period of this synthesized waveform to be the same as the period of a controlling source.

Used in this way, the generator becomes in effect a waveform <u>transformer</u>, operating at <u>real time</u>.

The Beauchamp Harmonic generator (see Chapter I) achieves its waveform by using a hetrodyning technique at a high frequency, producing a system of harmonically related frequencies all derived from one high frequency master oscillator. The more frequency components included, the more complex becomes the hetrodyning scheme; hence the limitation to six harmonics.

In synthesizing an arbitrary waveform, it is clear that more than the first six harmonics would be required. A different approach, then, was used in the AWG which is related to the time sampling method of digitizing waveforms. A long used technique is that of sampling an unknown waveform at many equally spaced time intervals. Each time element is thus assigned a discrete amplitude and the waveform is digitized.

Here essentially the reverse of this process is used. The period of the waveform to be synthesized is divided into 96 equal time elements and the value of the waveform amplitude at the center of each element is stored in a memory. This memory is then commutated by a 96 element sequence generator and the output obtained is a close approximation to the desired waveform. This waveform is then filtered to remove the discontinuities which occur between successive time elements. Loosely speaking, the result of this technique is to generate a sequence of "dots" and then with a filter, draw a smooth curve through these dots. The actual degree to which an <u>arbitrary</u> waveform may be thereby synthesized will be considered in the next chapter. The sequence generator is driven at 96 times the waveform period by a square wave obtained from a frequency multiplier whose input is a real sound source. This gives us the "real time" capabilities mentioned above. The price that must be paid for this flexibility is that amplitude and phase of the Fourier components may not be changed as a function of time. If we wish to charge one Fourier component, we must change all 96 memory elements, not just one.

The block diagram for the AWG appears in Figure 1. A brief description of the overall operation will be given here while a more detailed discussion of each part will be saved for the next chapter. An audio input of frequency f_o is fed into a linear frequency to voltage converter. The conversion is such that the output voltage is proportional to the reciprocal of f_o. The reason for this will be made clear later on. The output of the converter k_1/f_o drives a voltage controlled oscillator whose conversion constant is $1/k_1$. Thus the output is $f = 1/f_o$ or the reciprocal of the input frequency. The process is repeated once more but with the VCO producing $f' = 96\ f_o$. The accuracy of the whole frequency multiplication process is desired to be on the order of 0.1%. As it is thought that each of the four elements involved in the process might have an error of about 0.1%, a feedback loop is used to correct for the compounded error. The response time of the multiplier is essentially instantaneous but with a phase shift of two cycles. (This will be discussed later.) f' now drives a $\div$ 96 scaler which in turn drives a diode matrix or sequence generator. The output of the scaler also drives the frequency comparitor in the feedback loop. If $f'/96$ differs from f_o averaged over T then the first converter is corrected in the proper direction.

The outputs of the diode matrix (96 lines) are sequentially true, one at a time at a switching rate of f'. The sequence repeats at the rate $f'/96$. These lines feed 96 potentiometers (see Figure 2). The potentiometers are the memory elements which are loaded by the operator by setting each of them to an amplitude as monitored on the panel meter. When a given matrix output line is true, the corresponding potentiometer amplitude is gated through to the or-gate. The output of the or-gate is then the approximated waveform or "dot pattern". This drives simultaneously a set of low pass filters with cutoff frequencies spaced one octave apart throughout the audio spectrum. We have only to gate the output of the proper filter which will remove only the switching discontinuities from the waveform at the particular frequency occurring at that moment. This is done by level discriminators monitoring the input voltage to the last VCO. The audio output will, then, be the waveform stored on the 96 potentiometers and will have a frequency f_o.

IV. DETAILED DESIGN CONSIDERATIONS

A. Frequency Tracking Circuit: Frequency Multiplier

Included in the general description in Chapter III was a system for multiplying an input frequency by 96. This result is used to drive the ÷ 96 scaler thereby producing a frequency at the output of the AWG which is equal to the input frequency. In order for this scheme to be worthwhile as a "real-time converter" the output must follow the input in frequency within a fairly short response time. It is desirable that this response time be short enough to track discrete frequency changes by the input and have them be perceived as at least nearly discrete changes at the output. Since pulses occurring at 20 cps or faster are perceived as a "tone" rather than a string of pulses, it is reasoned that a response time of 1/20 sec might be the limit of resolution of individual events by the ear due to the "aural integration" time noted in Chapter II. (For low frequencies and/or small frequency changes, the ear probably does not respond nearly this rapidly.)

It was decided that sufficient musical flexibility would be obtained if the AWG had a range of 40 cps to 4 kc. Thus, if we adopt 1/20 sec as the maximum response time of our instrument, for 40 cps, the frequency change would have to occur within approximately two cycles. This is expecting a great deal as will become evident in the following section.

Conventional Methods

One method of frequency multiplication which is commonly used is shown in block form in Figure 3. A squaring circuit consisting of perhaps a differential comparitor referenced to zero followed by a Schmidt Trigger would provide a change of state at each zero crossing. Provided that the input waveform had no more than two zero crossings per cycle (here we assume that the waveform is adjusted so that its average dc level is zero) the output would be a square wave at the fundamental frequency. The following one-shot should have a period just less than the period of the maximum input frequency; in our case, 1/(4 kc) = 250 μsec. If now a low pass filter could pass only the average dc component of the binary waveform at the one-shot output, we would have a linear frequency to voltage converter. (The voltage would be proportional to the number of zero crossings occurring in a given time interval.) A linear VCO could be driven from this source and the constant n could be adjusted such that f_2 is any multiple of f_1 desired. The response time with which f_2 follows f_1 is determined by the impulse response characteristics of the low pass filter. (At low frequencies, the 250 μsec pulse is essentially an impulse and at higher frequencies, the response may be found from the impulse response by the convolution integral.)

Two types of filters are commonly used in this circuit: a simple multi-section R-C integrator and a pi-section L-R-C filter. The impulse response of a two section R-C integrator is derived in the appendix and is plotted in Figure 4. The response is given by

$$v_2 = \frac{1}{2.24\ RC}\left(e^{-.38t/RC} - e^{-2.62t/RC}\right).$$

We see that the rise time is much faster than the decay, the peak occurring in about 2/3 RC and decaying asymptotically to $e^{-2.62t/RC}$. By superposition, we can infer the behavior of this network for a pulse train. Equilibrium will be reached when, during one period, the combined decay of all previous pulses is equal to the rise of the new pulse and this equilibrium value is the dc component fed to the VCO. A ripple will occur at the period of the impulse train, since the combined rise and fall do not completely cancel over all time. Since ripple could not be tolerated in the frequency control voltage, the time constant must be made long, or more sections added, to minimize the percentage of ripple. However, this is directly opposed to our objective in this case since the rate at which the network could respond to a change in frequency, i.e., reach a new equilibrium voltage, is the decay slope shown in Figure 4. This type of network cannot possibly respond to changes within 1/20 sec and still produce a smooth dc component at 40 cps.

The impulse response for a single section pi-network low-pass filter is also derived in the appendix. It is more complex than the above and is of the general form:

$$v_2 = Ae^{-t/RC} + e^{-\alpha t}\left(Be^{\beta t} + Ce^{-\beta t}\right)$$

where α is real and β may be real or imaginary, depending upon the Q of the inductor and the filter termination. If α is made small while β is imaginary, the filter "rings". For all but very light loading however, α is large and the last term damps out quickly, which would seem to be desirable in our case. This leaves us with a pure exponential term with the same problems as for the R-C integrator. Obviously, this is not a particularly good filter for our purposes either. Since no other filter that would serve our purposes was known to the author, this method of frequency multiplication was abandoned.

The second commonly used frequency multiplication scheme is shown in block form in Figure 5. The squared output from the zero crossing sensor is this time used to excite a singularity function such as a square wave or an impulse train and the bandpass filter will pick off the desired harmonic. This output (which is essentially a sine wave) is then fed into a saturating amplifier-shaper circuit. The band width of the band pass filter must be narrow enough to pass only one harmonic while wide enough to allow the necessary frequency deviation, $2\Delta f$, at the harmonic frequency. For our case this would be possible only if the input were hetrodyned up to a very high frequency, the 96th harmonic of the combination picked off and then hetrodyned back down by beating with the 96th harmonic of the original hetrodyning oscillator. Although this is technically a difficult enough problem to make this method of questionable value, there is even another more serious problem. When multiplying by large numbers, the singularity function period is enough longer than the natural response of the filter that only the impulse response can be excited no matter what the input function is. The filter will then "ring" at its natural frequency between impulses. Since the natural frequency of the filter is a characteristic of the filter and not the input, we have lost the objective. It is doubtful whether this method would work for greater multiplication factors than, say 20.

Computational Method

An entirely different approach which eliminates the need for a filter is the one indicated in Figure 1 and briefly outlined in Chapter III. The technique differs from conventional methods in that the analogue VCO input is "computed" every two cycles. The system devised, however, computes a voltage proportional to the period rather than the frequency so that the multiplication must be made in two steps with two VCO's.

The technique for obtaining the period to voltage conversion is shown in block form in Figure 6, and a corresponding timing chart is found in Figure 7. The input is squared as before but this time is divided by two giving us the two cycle sequence. This sequence is divided into three periods as can be seen in Figure 7. A linear integrator is turned on with one state of the scaler. At the end of exactly one period, the integration is stopped and the 100 μsec. DF is triggered. During the period of this DF, the gate is opened and the voltage, V_o, is fed through a current amplifier to a very large capacitor, C, which charges toward this value. At the end of the 100 μsec. period, the FF is set and the integrator is zeroed. At the end of the second input period, this FF is reset and the sequence begins again.

The value of the only fixed time element, 100 μsec., was chosen so as to leave a minimum of 150 μsec. for the zeroing to occur (at 4 kc). This process would involve removing the charge from a capacitor and must take a finite length of time. In a trial circuit (see Chapter VI), 150 μsec. was found to be necessary for the decay to become sufficiently close to zero. If this minimum is accepted, this results in a fixed period of 100 μsec in which to charge the capacitor C. The capacitor, C, is a memory device, storing the value of V_o between "updating" periods. C must be chosen so that the charging time constant is seven times less than 100 μsec and the discharge time constant (through R_L, the VCO input resistance) is at least 500 times the maximum interval between updating periods, or ~ 25 sec. These conditions will result in a 0.1% error in charging ($e^{-7} \approx .001$) and a 0.1% error in holding V_o for .05 seconds [$(1 - e^{-.002}) \approx .998$; avr. error = 0.1%]. Hence,

$$R_L C = 25 \text{ sec, } \underline{\text{min}}$$

$$R_S C = 14 \ \mu\text{sec, } \underline{\text{max}}$$

where R_S is the effective charging source resistance.

Assuming that R_L can be made as large as 5 Meg, the first condition fixes C.

$$C = 5 \text{ Mfd.}$$

The value of R_S is then specified by the second condition:

$$R_S \cong 3 \, \Omega .$$

If we design for V_o to take on the values of 0.05 to 5.0 volts, the maximum change in V_o which could occur in one charging period is ~ 5 volts. (This is extremely unlikely since this would mean a frequency change from 4 kc to 40 cps within .05 sec.) In this case, the source would have to be capable of supplying a current of

$$I = 5/3 = 1.67 \text{ Amps (max)} .$$

This is quite possible since the duty cycle would be low.

As was mentioned in Chapter III, it is thought that the other elements of the frequency multiplier, i.e., the VCO's could also be kept within 0.1% error. Assuming the worst case, where all the errors were additive, we will still have something less than a total of 1% error. Since we desire an overall error of 0.1% we may use a correction feedback loop. The error sensing element (or frequency comparitor) could utilize the pulse averaging technique discussed earlier in this chapter since a long time constant could be tolerated. The length of this time constant will ultimately have to be determined by use of the instrument but it is thought that 1 sec might be tolerable since for small changes in frequency the nonlinearity will be quite small and for large changes, the ear will not be as quick to sense the error.

The frequency multiplication circuits for the AWG as well as the filter switching circuits are not completed as of the writing of this thesis. However, it is felt that sufficient proof of its feasibility has been presented and its completion is planned in the near future. Some preliminary circuit design has been done however and will be discussed in Chapter VI.

B. <u>Waveform Synthesizer</u> (Dot Generator)

Figure 8 shows this portion of the AWG, which has been completed and is operating very satisfactorily. (Slightly expanded detail is shown here, compared to Figure 1.) For a general description of its operation, see Chapter III. The design considerations and circuits are presented below, while the evaluation of its operation appears in the next chapter.

<u>Scaler</u>

It was decided at the outset that about 100 elements were desired in the time division of the waveform period. Since the next largest power of 2 is 128, feedback would have to be employed in the scaler. Feedback in a scaler <u>must</u> occur a finite time after the normal switching transient and thus will shorten the period of the following time element by this time delay. It therefore seemed desirable to minimize the amount of feedback necessary and a scale of 96 was chosen, rather than 100, since this requires only one pulse to be fed back (see Figure 8). When the 2^7 scaler has reached count number 64 (half full, or 0000001) the switching transient is fed back (delayed by propagation time) to the next to last scaler, setting it to "1". The scaler then has a count of 96 stored in it (0000011) and continues to count toward 127. It will take 32 more inputs to return to zero; hence the scaler has really divided by 64 + 32, or 96. Note that in the feedback loop (see schematic, Figure 9) the 2N914 transistor has deliberately been made "sluggish" by placing a 75 μμfd capacitor from its base to ground thus increasing the propagation delay of the fed back pulse to a safe amount (~ 100 n sec).

Since the maximum driving frequency to the scaler will be 384 kc (96 x 4 kc) the scaler was designed to operate at a maximum frequency in excess of 1 Mc to allow a margin of safety. It is also necessary that the main switching transient propagate through all seven scalers in a small fraction of one time element period. Both of these objectives are met by using high-speed silicon switching transistors, 2N914's, in a high current FF circuit. The design of the FF is quite straightforward except that all the impedance elements are scaled to a much lower value than usual, resulting in smaller time constants and much faster rise times. For sake of reliability, the very conservative β of 10 was assumed for each transistor. The resulting power supply drain is quite large but the low impedance made it possible to use cheap germanium diodes in the diode matrix which greatly cut the cost of the device.

In order to prevent degradation of rise times and propagation delay, isolation between the scaler circuits and the diode matrix is provided by two amplifiers, a 2N966 high speed germanium and a 2N3053 high current silicon switch. Since the emitter-base junction of a germanium transistor must be driven into the reverse region in order for the transistor to be completely cut off, the emitters of all the 2N966 are returned to a 9 volt supply rather than +10. This will just a little more than compensate for the voltage divider effect of the base and collector resistors, preventing the FF collectors from rising all the way to +10 volts. The 2N966 is an inverting saturating amplifier which drives the 2N3053 emitter follower current amplifier. The maximum current supplied to the diode matrix by any given

2N3053 (see Appendix) is 63 Ma. This places it in the range of maximum β which is typically ~ 100 [RCA transistor data sheet, 1963] so the isolation should be good. To increase dissipation capabilities of these transistors, a common heat sink bar was used. (The collectors are tied to the transistor can.) [See Figure 10.]

Sequence Generator and Memory

Figure 11 shows schematically one element of the sequence generator and memory circuits. There are 96 such elements. The operation of the matrix is most easily seen if we realize that it is made up of a series of and-gates, one for each possible binary combination of the scaler FF's. In our case, there are 96 possible binary combinations and, hence 96 and-gates in the matrix. The seven inputs of each and-gate are connected to the unique permutation of the 14 scaler outputs which correspond to that and-gate's place in the sequence. When, and only when all seven inputs are true (0 volts), the output (2N404 base) will be allowed to fall to zero. In all other 95 cases, the 2N404 base will be driven to ~ +9 volts by one or more of the 7 inputs. The emitter of all 96 2N404's are tied together and to a common load resistor (to +10). This forms a high input impedance or-gate. The memory elements consist of 5K pots providing an adjustable (by the operator) supply voltage to the emitter follower or-gate elements. When the 2N404 base is at ~ 0 volts, it is biased well into saturation, regardless of the setting of the pot. Thus, the emitter is pulled down to the potentiometer potential. When the 2N404 base is at about +9 volts, it is well cut off by virtue of the fact that some other 2N404 is saturated, forcing the common emitters to be at a value between +1 and +6 volts. Hence, as each of the and-gate outputs sequentially becomes true, the corresponding potential stored on the potentiometer is gated to the output. The driver amplifier shown in Figure 8 at the or-gate output is simply an emitter follower current amplifier to provide a low impedance for the filter inputs.

Resolution

The accuracy with which an arbitrary periodic waveform may be represented by 96 sequential discrete amplitude pulses may be considered in terms of how many harmonics may be resolved with negligible error. For example, if the 16th harmonic were present, there would be 6 time elements in its period. If we could filter out all the overtones thereby generated, we would be able to synthesize arbitrary waveforms to within the 16th harmonic. Let us consider this case further. Figure 12 shows the synthesized 16th harmonic. The maximum amplitude will be considered 1 for simplicity. The value of sin t at the center of each element has been assigned for the length of that element. The Fourier series expansion of such a waveform is found to be:

$$f(t) = 3/\pi [\sin t + 1/5 \sin 5t + 1/7 \sin 7t + 1/11 \sin 11t + 1/13 \sin 13t + \dots].$$

By using a pi-network low-pass filter which has a rolloff of ~ 60 db/decade (see Figure 13) and a cutoff at $\omega = 1$, the first error term would be down by ~ 50 db + 20 log 5 = 64 db. However, we wish to place these filters only one octave apart so that this same filter must perform for a wave of 1/2 the frequency of the above; hence with a first error term of 1/5 sin 5/2 t. The attenuation in this case would be ~ 30 db + 20 log 5 = 44 db. This is deemed to be a just satisfactory amount of attenuation and thus the 16th harmonic has been specified as the maximum resolution of the AWG. Perhaps more than 16 harmonics could be resolved using filters with sharper cutoffs, but since there are nine of these filters, it was decided that they be kept as simple as possible.

Filters

The nine low-pass filters whose response characteristics have already been introduced in Figure 13 are shown in schematic form in Figure 14. The basic circuit design for all the filters is the same, built around the 600 ~ filter (0.068 μfd, 2 Hy., 10 K) the others being scaled in frequency. In some cases, impedance scaling was also used to make possible the use of inductors on hand.

The emitter follower driver amplifier has an effective impedance low compared to the filter input impedance so that it does not become a part of the filter circuit (i.e., it appears as an ideal voltage source). All nine filters are driven in parallel and the output may be selected from the proper filter. Presently, a selector switch is used for the output switching since the automatic circuit is not completed.

Loading Memory

Loading the memory with the desired waveform is effected by setting each of the 96 potentiometers to the value of the waveform at the center of the corresponding time element. A meter, calibrated with a 100-0-100 scale, is connected to the dc output of the or-gate and is used as a monitor for setting these values. For Fourier series synthesis, see charts in Appendix and discussion in Chapter VII.

Provision must be made to operate the scaler in a "push-button-advance" mode during the loading operation. Each time element can be held until the correct reading is obtained on the meter and then the push button causes the scaler to advance to the next element. This is done by the circuit shown in Figures 15 and 16. During normal cycling, the switch is in "operate" position. This clamps the FF to the set state and allows the square wave to pass through the and-gate and hence to the scaler. When the switch is put into "LOAD" position, the FF is not reset immediately but the reset input is connected to the ÷ 96 scaler output. When the scaler resets, the FF resets, closing the square wave gate and opening the push button gate. The scaler is thus stopped in the reset position and will remain there until advanced by the push button. The purpose of the two DF circuits is to provide isolation from the "noisy" push button switch closures. The first DF has a period long enough that a bouncing closure of the switch can cause it to be triggered only once since by the end of its period, the input transients have died out. The second DF provides a uniform pulse as an input to the scaler in addition to providing additional noise discrimination.

The schematic shows that straightforward logic circuits have been used throughout. Diode and gates were used while the or-gate is a saturating amplifier type to regain any degradation of rise times throughout the gating circuit.

Filter Switching

It is planned that when the frequency tracking portion of the AWG is completed, the filters may be switched through a set of discriminators operating from the VCO control voltage. The discriminators will probably be differential comparitors with adjustable references so that the cross-over frequencies may be "pin-pointed". Although this detail was not shown in Figure 1 (for sake of simplicity) an exclusive or-gate between each successive pair of discriminators would provide the correct control voltage to the filter gates. If one input is a "1" the output is a "1", but when two successive discriminators give an output of "1" the exclusive or goes to "0". However, the next exclusive or-gate becomes true and so on, enabling the filter gates in succession.

The filter gates, themselves, present somewhat of a problem. Preliminary experiments show that at least ordinary germanium transistors used in either a shunt-gate or series-gate configuration, show a feed-through on the order of 40-50 db. This probably is not tolerable since unwanted harmonics from the higher cut-off filters would be allowed to leak through by this amount. It is possible, however, that this figure could be improved somewhat with the use of chopping transistors which are made specifically to have a low leakage in saturation. Should these still not prove satisfactory, relay switching would be the answer, although the extra driving circuitry makes them a second choice.

V. EVALUATION OF SYNTHESIZED WAVEFORMS

Evaluation of the synthesizer portion of the AWG turned out to be more difficult than originally anticipated. The spectrograms made showed conflicting results and in many cases were not repeatable. Thus, the spectrum analysis of the synthesized waveforms which was expected to be most conclusive had to be discarded.

The device used to make the amplitude vs frequency plots was a Kay Electronics Missilizer, Model 675 . The failure to obtain meaningful results is mainly attributed to the link between the AWG and the Missilizer. Since the two instruments are located in different buildings, the waveforms were recorded on an Ampex Model 350 magnetic tape recorder and transported to the Missilizer where they were reproduced on an Ampex Model 302. Another recording operation occurs in the Missilizer itself. Nonlinearity was apparently introduced in one or more of these operations producing spectrum lines for almost all product terms as well as harmonics which were only a few db down from the actual spectrum lines. Some nonlinearity in the waveform is undoubtedly produced by the nonlinearity of the meter in the AWG. This is specified by the manufacturer to be less than 2% however and would not account for the results obtained. One spectrogram of a filtered sine wave showed spectral components in the cutoff region of the low-pass filter which would indicate that the unfiltered waveform contained harmonics 30 db stronger than the fundamental! We therefore conclude that the distortion is occurring after the low-pass filter which is the output of the AWG.

Since a harmonic wave analyzer was not available, perhaps the most sensitive evaluation of the synthesized waveforms can be made by ear. Comparisons between waveforms generated by the four sine generators in the lab and synthesized waveforms showed that they were nearly indistinguishable. Some perceptual difference is to be expected due to the non-synchronized phase relationships of the four generators.

Another somewhat qualitative evaluation of the waveforms can be made by observing the waveforms on an oscilloscope. Figures 17 and 18 show photographs of synthesized waveforms displayed on a Heathkit Model IO-12 scope. The amplitude settings for these waveforms was obtained from the computer charts found in the appendix. Figure 17 shows unfiltered and filtered forms of sin x (96 dots per cycle) and sin 16x (6 dots per cycle). Both appear to be excellent approximations to sine waves. (With extreme concentration, the second harmonic of the sin 16x waveform can just be heard.) Examples of more complex waveforms are shown in Figure 18. The complexity of these waveforms seems to defy quantitative evaluation by any means of measurement available to the author. The product terms are very prominent (as they should be) as perceived by the ear and one would find it difficult to identify any spurious frequencies. The photographs however testify that the filtered waveforms are indeed smooth curves through the "points" of the unfiltered waveforms.

These results are representative of the performance obtained throughout the 40 cps-4kc range of the instrument. An "endless" variety of waveforms can of course be produced either by Fourier synthesis using the charts in the appendix or by "drawing" a waveform on the oscilloscope screen with the potentiometers. Rather simple examples were shown here to make analysis uncomplicated.

Since the waveforms produced are steady states, rather accurate spectral analysis probably could be obtained with a wave analyzer such as a Hewlett-Packard Model 302A. It is hoped that in the future such an instrument may be available for the evaluation of the synthesizer.

VI. PRELIMINARY AND PROJECTED DESIGN OF FREQUENCY MULTIPLICATION CIRCUITRY

The accuracy required from the frequency multiplication circuits has made it difficult to test and evaluate individual elements with the limited equipment available in our lab. Although the design and construction of these circuits have not been completed as of this writing, it is believed that the preliminary work to be discussed below shows that such a system is feasible. It is hoped that this work can be completed in the near future.

Blocks 1-4 in Figure 6 are quite conventional circuits and will not be discussed here. It should be noted, however, that high speed switching transistors must be used throughout these circuits in order to keep propagation times negligible.

Two different circuits might be used for the linear integrator (block 5), a "boot-strap" ramp generator or an operational amplifier integrator. The former, which is the simpler of the two, was constructed and is shown schematically in Figure 19. Connected to the circuits of blocks 2, 3, and 4 and a square wave generator, this circuit produced the desired waveform, 5, in Figure 7.

In order to generate the linear ramp portion of the waveform, a constant current source charges the 3 ufd capacitor. Q_4 is used to "zero" the circuit and is normally cut off. Q_2 is saturated during integration and is cut off during the "hold" period. During the zeroing period, C_2 is forced to charge to the reference voltage through the 1N90 diode. When the "zero" input goes neg, C_1 begins charging with the current through the 15k resistor. The potential across this 15k resistor is kept constant by the now "floating" 68 ufd capacitor and Q_3, an emitter follower. If Q_3 has a β of about 50, the current discharging C_2 is about 1/50 of that charging C_1 and since C_1 is about 1/20 of the value of C_2, the current charging C_1 is constant within $1/50 \cdot 1/20 = 1/1000$. For a reference of 10 volts, $I_c = 2/3$ Ma. The slope of the ramp is then I_c/C_1 since:

$$V = \frac{1}{C_1} \int_o^t I_c \, dt = \frac{I_c}{C_1} t$$

$$\frac{dv}{dt} = \frac{I_c}{C_1} = \frac{.67}{3} \times 10^3 = 220 .$$

Thus V_o for a period of 1/40 sec = 5.5 volts and for 1/4000 sec = 0.055 volts. By varying the reference voltage, the slope may be changed. This provides a convenient input from the error correction feedback loop. At the end of the "integrate" period Q_1 and hence Q_2 is cut off and negligible current flows into C_1 leaving V_o at the output until the zero period begins. (Note, Q_2 must have a low I_{CBO} leakage current and a PNP silicon should be used in the final circuit.) The only test of performance of this circuit to date was simply observation of the waveform shape versus frequency on a Heathkit IO-12, AC-coupled oscilloscope. The waveshape appeared to be quite good but not much accuracy can be claimed for this test. When the entire system of Figure 6 is completed, the system may easily be evaluated as a whole with a d.v.m.

Should this ramp generator not prove adequate, a slightly more complicated but straightforward circuit shown in Figure 20 might be used. The operational amplifier connected as shown behaves as a linear integrator, the transfer function being:

$$V_{OUT} = -\frac{1}{RC}\int_0^t V_{IN}\,dt$$

assuming A is very large and $V_{IN}(0) = 0$ [Hartley, 1962]. We may achieve almost any accuracy from this circuit by making A large enough. There is no leakage problem during the "hold" period for we simply need to keep the input at zero volts. The zeroing of C would be performed in much the same way as in the bootstrap circuit.

The circuit used to store V_o on the "memory capacitor", C in Figure 6, can be a system of emitter followers. A technique shown in Figure 21 would allow current amplification without a shift in dc level. V' will be above V_{IN} by the emitter-base junction forward drop of Q_1. V_{OUT} will be below V' by the emitter-base junction forward drop of Q_2. If Q_1 and Q_2 are both silicon these two forward drops cancel and $V_{OUT} \simeq V_{IN}$. The same occurs in reverse order in Q_3 and Q_4 and V_{OUT} is obtained through both paths. The complementary output gives the ability to supply charging or discharging current, whichever happens to be needed. More stages than are shown in Figure 21 will be required to give the current gain necessary to allow a maximum output of 1.67 amps, while drawing no more than about 3 μA from the 3μfd capacitor. (This value gives .1% discharge error for 4 kc.)

The gate presents somewhat more of a problem, that of keeping leakage currents to a negligible value while the gate is off. A complementary series gate at an intermediate level in the current amplifier would probably suffice. The complementary configuration would provide a slight cancellation of leakage currents as well as supplying current in both directions.

VCO circuits with 0.1% linearity are readily found in the literature and should not present serious problems. Voelker [1964] published a suitable circuit using a bootstrap amplifier driving a unijunction transistor relaxation oscillator. Another circuit using an operational amplifier-unijunction oscillator combination appears in the GE Transistor Manual [Seventh Ed., 1964].

VII. RELATED COMPUTER PROGRAMS AND CONTROL

Setting the 96 potentiometers on the AWG requires that the operator know the value of the function he wishes to generate at 96 equally spaced points over one period. If the function is not known in analytical form, one may obtain these values through graphical methods. Frequently, however, one would like to specify a sound in terms of its Fourier components. If charts of sin (nx) were available where x took on units of $2\pi/96$ over 2π, the final waveform values could be found by adding the components from the charts. Since these charts were not available, a simple Fortran program was written for their generation. The Fortran source list is shown in Figure 22 and the charts are found in the Appendix. The source list shown will generate the 16 pages directly and requires no input data. Each page is labeled according to the harmonic order of the data printed thereon. The data are listed in tabular form with the amplitude appearing opposite a channel number, one for each of the 96 knobs on the memory panel. At the bottom of each chart is phase shift information for the corresponding Fourier term. With the information provided and a desk calculator, one can find the 96 input values for any Fourier sum with $n \leq 16$. After summation, one must divide all 96 values by either the sum of the Fourier coefficients (this will not normalize to 1.0 but will guarantee that no number will exceed 1.0) or the largest number obtained in the summation.

The above procedure is somewhat lengthy and tedious. It is for this reason that a program has been written which produces normalized input data for the AWG from a deck of input cards specifying the Fourier coefficients and phase shifts. A flow chart for this program is shown in Figure 23. Many different waveforms may be handled in one run, the output data being tabulated in charts similar to those in the program of Figure 22. This program is actually just an extension of the earlier one, the first operation being the generation and storage of the chart data. The entire 16 pages, in matrix form, occur twice in storage in such a manner that a simple change in the I index results in a phase shift of the corresponding Fourier component. A waveform is specified by one or two cards, each of which is divided into 9 fields. A field specifies the order, amplitude, and phase of a component of the waveform. Each waveform is separated from preceding and following waveform data by a blank field. A card with N (K) > 16 would function as a trailer and would end the run.

One further extension which is, however, somewhat expensive would provide automatic memory loading for the AWG. If the potentiometers were replaced by digital memory elements, i.e., FF's, the amplitude values could be entered from a paper-tape reader, card reader, or whatever such device might become available. In fact a very simple system is visualized which would allow data to be stored on 1/4-inch tape in the form of tone bursts and would load the AWG memory in about 1 minute. In this way, one of the tape machines already in the studio could function as a tape reader. The memory FF's in this case could be of a very inexpensive design, incorporating germanium transistors of the thirty cent variety. Each memory element would require eight FF's for a capacity of 256 (or an amplitude range of ± 128) which gives approximately the same resolution as the potentiometers and panel meter combination. Thus 768 FF's are required and would be connected in a shift-register configuration allowing a simple gating system. Data for many waveforms could be easily stored for future use on tape reels with voice identification before each set of tone bursts.

VIII. CONCLUSION

The Arbitrary Waveform Generator in its present form has already proven to be an interesting and useful instrument for the composer of electronic music. It has obvious limitations in that waveforms cannot be changed rapidly. It does, however, provide very accurate numerical control over waveshape parameters for the steady state while still remaining a relatively simple piece of equipment. This was the objective of this project and herein lies its value. The only other instrument (known to the author) capable of synthesizing arbitrary waveforms with equal accuracy is the digital computer itself.

When the frequency tracking portion of the AWG is completed, the advantages thereby gained should far overshadow the tedium of loading the memory. Operating as a waveform "transformer", the AWG should improve the composer's efficiency by allowing him to produce a long tone sequence in one operation. By experimenting with frequency modulation of the input waveform, new information may well be obtained as to the relative importance of this parameter.

Thus, it is thought that the AWG is a significant contribution to the electronic production of sound and we believe that it will find much use in the electronic music studios at this University.

APPENDIX A

Impulse Response of a Two Section R-C Integrator

The loop equations for a two section R-C integrator in the frequency domain where v_1 is the input and v_2 is the output are:

$$v_1 = I_1(R + 1/Cs) - I_2/Cs \quad (1)$$

$$0 = -I_1/Cs + I_2(2/Cs + R) \quad (2)$$

$$v_2 = I_2/Cs \quad (3)$$

For the impulse response of the filter, we let $v_1 = 1$ since an impulse transforms to the frequency domain as unity. Solving (1), (2), and (3) for v_2 and setting $v_1 = 1$, we obtain

$$v_2 = \frac{1}{R^2C^2}\left[\frac{1}{\left(s+\frac{.38}{RC}\right)\left(s+\frac{2.62}{RC}\right)}\right] \quad (4)$$

By partial fraction expansion, we find that (4) is equivalent to:

$$\mathcal{V}_2 = \frac{1}{2.24\,RC}\left[\frac{1}{\left(s + \frac{.38}{RC}\right)} - \frac{1}{\left(s + \frac{2.62}{RC}\right)}\right] \tag{5}$$

Equation (5) is in standard form and may be transformed to the time domain by the Laplace transform which yields

$$\mathcal{V}_2 = \frac{1}{2.24\,RC}\left[e^{-\frac{.38t}{RC}} - e^{-\frac{2.62t}{RC}}\right] \tag{6}$$

This is the impulse response of a two section R-C integrator and is plotted in Figure 4.

APPENDIX B

Impulse Response of Simple R-L-C Low Pass Filter

Figure 24 shows the schematic of a π-section low-pass filter. Although a specific example is shown, the impulse response will be derived for general component values and then applied specifically to the filter shown. Letting R_{in} and $R_{out} = R_o$ and R_L = the inductor resistance, we can write the loop equations in the frequency domain:

$$1 = I_1(R_0 + 1/Cs) - I_2/Cs \tag{1}$$

$$0 = -I_1/Cs + I_2(2/Cs + Ls + R_L) - I_3/Cs \tag{2}$$

$$0 = -I_2/Cs + I_3(1/Cs + R_0) \tag{3}$$

$$v_2 = I_3 R_0 \tag{4}$$

where the input voltage is an impulse and v_2 is the output. Solving for v_2 :

$$v_2 = \frac{R_0/C^2 s^2}{(R_0 + 1/Cs)^2 (2/Cs + Ls + R_L) - 2/C^2s^2 (R_0 + 1/Cs)} \tag{5}$$

Rewriting (5):

$$v_2 = \frac{R_0 C}{(R_0 C s + 1)\left[(R_0 L C^2) s^2 + (R_0 R_L C^2 + LC) s + (2R_0 C + R_L C)\right]} \tag{6}$$

The denominator in (6) may be factored:

$$v_2 = \frac{1}{(s + 1/R_0 C)(s + \alpha - \beta)(s + \alpha + \beta)} \tag{7}$$

where

$$\alpha = \frac{R_0 R_L C + L}{2 R_0 L C}$$

$$\beta = \frac{\sqrt{(R_0 R_L C)^2 + L^2 - 2R_0 L C (4R_0 + R_L)}}{2 R_0 L C}$$

Enpanding (7) by partial fractions, we obtain:

$$v_2 = \frac{1}{1/R_0^2 C^2 - \frac{2\alpha}{R_0 C} + \alpha^2 - \beta^2} \cdot \frac{1}{(s + 1/R_0 C)} + \frac{1}{2\beta^2 - 2\alpha\beta + 2\beta/R_0 C} \cdot \frac{1}{(s + \alpha - \beta)}$$
$$+ \frac{1}{2\beta^2 + 2\alpha\beta - 2\beta/R_0 C} \cdot \frac{1}{(s + \alpha + \beta)} \tag{8}$$

The Laplace transform of (8) is:

$$v_2 = \frac{1}{1/R_o^2C^2 - 2\alpha/R_oC + \alpha^2 - \beta^2}\, e^{-t/R_oC}$$

$$+ \frac{e^{-\alpha t}}{2}\left[\frac{1}{\beta^2 - \alpha\beta + \beta/R_oC}\, e^{\beta t} + \frac{1}{\beta^2 + \alpha\beta - \beta/R_oC}\, e^{-\beta t}\right] \quad (9)$$

This is the low-pass filter impulse response whose character is determined by the values of α and β. Both, however, depend on all four variables, R_o, R_L, L, and C, and it is difficult to make completely general statements about this response. It can be said, though, that α is always real and that for all but rather unrealistic values of R_o and C, α is large. β is generally imaginary but could be real for sufficiently small values of R_o (heavy loading). In the particular case where the inductor Q is high (R_L, small) and the loading is very light so that $R_oC \approx 1$, α will be small and the filter will "ring". For the circuit values shown in Figure 24, $\alpha \approx 70$, $\beta \approx j\,160$, and $1/R_oC \approx 100$. Hence, the second term is oscillatory at about $\frac{160}{2} \cong 25$ cps, but both terms damp out in about one cycle.

APPENDIX C

Fourier Expansion of sin (16x) Waveform

Figure 12 shows a sine wave approximated by six discrete square pulses. The Fourier expansion of this waveform will be derived here. The function is specified by:

$$
\begin{aligned}
f(x) &= \tfrac{1}{2}, \quad 0 < x < \pi/3 \\
&= 1, \quad \pi/3 < x < 2\pi/3 \\
&= \tfrac{1}{2}, \quad 2\pi/3 < x < \pi
\end{aligned}
\qquad (1)
$$

$f(x)$ is an odd function, periodic over 2π and can be expanded in terms of sine functions only.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(nx) \qquad (2)$$

where

$$b_n = 2/\pi \int_0^{\pi} f(x) \sin(nx)\, dx \qquad (3)$$

Evaluation of (3) gives

$$b_n = \frac{1}{n\pi}\left(1 + \cos(nx)\Big|_{\pi/3} - \cos(nx)\Big|_{2\pi/3} - \cos(nx)\Big|_{\pi} \right), \quad n = 1, 2, 3, \cdots \tag{4}$$

Equation (4) yields the following coefficients:

$b_1 = 3/\pi$	$b_5 = 3/5\pi$	$\vdots$
$b_2 = 0$	$b_6 = 0$	$b_{11} = 3/11\pi$
$b_3 = 0$	$b_7 = 3/7\pi$	$b_{13} = 3/13\pi$
$b_4 = 0$	$b_8 = 0$	$\vdots$

Hence, $f(x)$ may be written as

$$f(x) = \frac{3}{\pi}\left(\sin x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x + \frac{1}{11}\sin 11x + \frac{1}{13}\sin 13x + \cdots \right)$$

APPENDIX D

Diode Matrix Loading

In order to design the $\div$ 96 scaler circuit which drives the diode matrix it was necessary to know the amount of loading the matrix would present, i.e., the maximum current that would be drawn from each FF output. In an ordinary "square" matrix, there are 2^n input permutations and 2^n load resistors (since there are 2^n and-gates). There are 2n inputs of which n are supplying current at any time. There are 2^n outputs, only one of which is true at any time. Hence, all but one of the 2^n load resistors are always drawing current. Since the square matrix is completely symmetrical, we can simply conclude that the 2^n-1 load resistors are equally supplied by the n outputs for an average load of $2^{n-1}/n$.

In the case where feedback is used the matrix is no longer symmetrical and the above formula does not follow. The author was unable to derive the correct formula for a general matrix and finally resorted to tabular methods exhausting all possible loading conditions for each FF output. It was found that the load was not constant in time and varied among FF outputs. The results are tabulated below.

TABLE III

Matrix Loading

Output		Number of Loads	
FF No. 7	set output	10.5	max
		7.64	min
		6.05	avr
FF No. 7	reset output	18.14	max
		0	min
		6.05	avr
FF No. 6	set out	18.14	max
		0	min
		6.05	avr
FF No. 5	reset out	10.5	max
		7.64	min
		6.05	avr
FF[s] 1-5	set and reset outputs	14.8	max
		13.84	min
		7.08	avr

Since each load is 5.6 K and nearly 20 volts appears across them under load conditions the current per load is ≈ 3.5 Ma During the memory loading operation, the scaler will come to rest in all 96 positions for any length of time and hence, the outputs of the FF must be able to supply the maximum currents continuously. For this reason all FF outputs were assumed to have to supply ~ (18 x 3.5) Ma or 63 Ma.

APPENDIX E

Charts of sin (nx) up to n = 16

The following 16 pages contain values of sin (nx) where x takes on the values at the centers of 96 equal intervals between 0 and 2π (i.e., $x = \pi/96, 3\pi/96, 5\pi/96, \ldots$). Each page contains data for a different value of n from 1 to 16. At the bottom of each chart is the number of degrees (of nx) represented by each increment. The data were generated by the 7044 computer by means of the program listed in Figure 22.

SIN X = FUNDAMENTAL

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.03	33	0.85	65	-0.88
2	0.10	34	0.81	66	-0.91
3	0.16	35	0.77	67	-0.94
4	0.23	36	0.73	68	-0.96
5	0.29	37	0.68	69	-0.97
6	0.35	38	0.63	70	-0.99
7	0.41	39	0.58	71	-1.00
8	0.47	40	0.53	72	-1.00
9	0.53	41	0.47	73	-1.00
10	0.58	42	0.41	74	-1.00
11	0.63	43	0.35	75	-0.99
12	0.68	44	0.29	76	-0.97
13	0.73	45	0.23	77	-0.96
14	0.77	46	0.16	78	-0.94
15	0.81	47	0.10	79	-0.91
16	0.85	48	0.03	80	-0.88
17	0.88	49	-0.03	81	-0.85
18	0.91	50	-0.10	82	-0.81
19	0.94	51	-0.16	83	-0.77
20	0.96	52	-0.23	84	-0.73
21	0.97	53	-0.29	85	-0.68
22	0.99	54	-0.35	86	-0.63
23	1.00	55	-0.41	87	-0.58
24	1.00	56	-0.47	88	-0.53
25	1.00	57	-0.53	89	-0.47
26	1.00	58	-0.58	90	-0.41
27	0.99	59	-0.63	91	-0.35
28	0.97	60	-0.68	92	-0.29
29	0.96	61	-0.73	93	-0.23
30	0.94	62	-0.77	94	-0.16
31	0.91	63	-0.81	95	-0.10
32	0.88	64	-0.85	96	-0.03

EACH CHANNEL = 3.75 DEGREES

SIN 2X = 2ND HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.07	33	-0.90	65	0.83
2	0.20	34	-0.95	66	0.75
3	0.32	35	-0.98	67	0.66
4	0.44	36	-1.00	68	0.56
5	0.56	37	-1.00	69	0.44
6	0.66	38	-0.98	70	0.32
7	0.75	39	-0.95	71	0.20
8	0.83	40	-0.90	72	0.07
9	0.90	41	-0.83	73	-0.07
10	0.95	42	-0.75	74	-0.20
11	0.98	43	-0.66	75	-0.32
12	1.00	44	-0.56	76	-0.44
13	1.00	45	-0.44	77	-0.56
14	0.98	46	-0.32	78	-0.66
15	0.95	47	-0.20	79	-0.75
16	0.90	48	-0.07	80	-0.83
17	0.83	49	0.07	81	-0.90
18	0.75	50	0.20	82	-0.95
19	0.66	51	0.32	83	-0.98
20	0.56	52	0.44	84	-1.00
21	0.44	53	0.56	85	-1.00
22	0.32	54	0.66	86	-0.98
23	0.20	55	0.75	87	-0.95
24	0.07	56	0.83	88	-0.90
25	-0.07	57	0.90	89	-0.83
26	-0.20	58	0.95	90	-0.75
27	-0.32	59	0.98	91	-0.66
28	-0.44	60	1.00	92	-0.56
29	-0.56	61	1.00	93	-0.44
30	-0.66	62	0.98	94	-0.32
31	-0.75	63	0.95	95	-0.20
32	-0.83	64	0.90	96	-0.07

EACH CHANNEL = 7.50 DEGREES

SIN 3X = 3RD HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.10	33	0.10	65	0.10
2	0.29	34	0.29	66	0.29
3	0.47	35	0.47	67	0.47
4	0.63	36	0.63	68	0.63
5	0.77	37	0.77	69	0.77
6	0.88	38	0.88	70	0.88
7	0.96	39	0.96	71	0.96
8	1.00	40	1.00	72	1.00
9	1.00	41	1.00	73	1.00
10	0.96	42	0.96	74	0.96
11	0.88	43	0.88	75	0.88
12	0.77	44	0.77	76	0.77
13	0.63	45	0.63	77	0.63
14	0.47	46	0.47	78	0.47
15	0.29	47	0.29	79	0.29
16	0.10	48	0.10	80	0.10
17	-0.10	49	-0.10	81	-0.10
18	-0.29	50	-0.29	82	-0.29
19	-0.47	51	-0.47	83	-0.47
20	-0.63	52	-0.63	84	-0.63
21	-0.77	53	-0.77	85	-0.77
22	-0.88	54	-0.88	86	-0.88
23	-0.96	55	-0.96	87	-0.96
24	-1.00	56	-1.00	88	-1.00
25	-1.00	57	-1.00	89	-1.00
26	-0.96	58	-0.96	90	-0.96
27	-0.88	59	-0.88	91	-0.88
28	-0.77	60	-0.77	92	-0.77
29	-0.63	61	-0.63	93	-0.63
30	-0.47	62	-0.47	94	-0.47
31	-0.29	63	-0.29	95	-0.29
32	-0.10	64	-0.10	96	-0.10

EACH CHANNEL = 11.25 DEGREES

SIN 4X = 4TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.13	33	0.79	65	-0.92
2	0.38	34	0.61	66	-0.99
3	0.61	35	0.38	67	-0.99
4	0.79	36	0.13	68	-0.92
5	0.92	37	-0.13	69	-0.79
6	0.99	38	-0.38	70	-0.61
7	0.99	39	-0.61	71	-0.38
8	0.92	40	-0.79	72	-0.13
9	0.79	41	-0.92	73	0.13
10	0.61	42	-0.99	74	0.38
11	0.38	43	-0.99	75	0.61
12	0.13	44	-0.92	76	0.79
13	-0.13	45	-0.79	77	0.92
14	-0.38	46	-0.61	78	0.99
15	-0.61	47	-0.38	79	0.99
16	-0.79	48	-0.13	80	0.92
17	-0.92	49	0.13	81	0.79
18	-0.99	50	0.38	82	0.61
19	-0.99	51	0.61	83	0.38
20	-0.92	52	0.79	84	0.13
21	-0.79	53	0.92	85	-0.13
22	-0.61	54	0.99	86	-0.38
23	-0.38	55	0.99	87	-0.61
24	-0.13	56	0.92	88	-0.79
25	0.13	57	0.79	89	-0.92
26	0.38	58	0.61	90	-0.99
27	0.61	59	0.38	91	-0.99
28	0.79	60	0.13	92	-0.92
29	0.92	61	-0.13	93	-0.79
30	0.99	62	-0.38	94	-0.61
31	0.99	63	-0.61	95	-0.38
32	0.92	64	-0.79	96	-0.13

EACH CHANNEL = 15.00 DEGREES

SIN 5X = 5TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.16	33	-0.94	65	0.77
2	0.47	34	-1.00	66	0.53
3	0.73	35	-0.96	67	0.23
4	0.91	36	-0.81	68	-0.10
5	1.00	37	-0.58	69	-0.41
6	0.97	38	-0.29	70	-0.68
7	0.85	39	0.03	71	-0.88
8	0.63	40	0.35	72	-0.99
9	0.35	41	0.63	73	-0.99
10	0.03	42	0.85	74	-0.88
11	-0.29	43	0.97	75	-0.68
12	-0.58	44	1.00	76	-0.41
13	-0.81	45	0.91	77	-0.10
14	-0.96	46	0.73	78	0.23
15	-1.00	47	0.47	79	0.53
16	-0.94	48	0.16	80	0.77
17	-0.77	49	-0.16	81	0.94
18	-0.53	50	-0.47	82	1.00
19	-0.23	51	-0.73	83	0.96
20	0.10	52	-0.91	84	0.81
21	0.41	53	-1.00	85	0.58
22	0.68	54	-0.97	86	0.29
23	0.88	55	-0.85	87	-0.03
24	0.99	56	-0.63	88	-0.35
25	0.99	57	-0.35	89	-0.63
26	0.88	58	-0.03	90	-0.85
27	0.68	59	0.29	91	-0.97
28	0.41	60	0.58	92	-1.00
29	0.10	61	0.81	93	-0.91
30	-0.23	62	0.96	94	-0.73
31	-0.53	63	1.00	95	-0.47
32	-0.77	64	0.94	96	-0.16

EACH CHANNEL = 18.75 DEGREES

SIN 6X = 6TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.20	33	0.20	65	0.20
2	0.56	34	0.56	66	0.56
3	0.83	35	0.83	67	0.83
4	0.98	36	0.98	68	0.98
5	0.98	37	0.98	69	0.98
6	0.83	38	0.83	70	0.83
7	0.56	39	0.56	71	0.56
8	0.20	40	0.20	72	0.20
9	-0.20	41	-0.20	73	-0.20
10	-0.56	42	-0.56	74	-0.56
11	-0.83	43	-0.83	75	-0.83
12	-0.98	44	-0.98	76	-0.98
13	-0.98	45	-0.98	77	-0.98
14	-0.83	46	-0.83	78	-0.83
15	-0.56	47	-0.56	79	-0.56
16	-0.20	48	-0.20	80	-0.20
17	0.20	49	0.20	81	0.20
18	0.56	50	0.56	82	0.56
19	0.83	51	0.83	83	0.83
20	0.98	52	0.98	84	0.98
21	0.98	53	0.98	85	0.98
22	0.83	54	0.83	86	0.83
23	0.56	55	0.56	87	0.56
24	0.20	56	0.20	88	0.20
25	-0.20	57	-0.20	89	-0.20
26	-0.56	58	-0.56	90	-0.56
27	-0.83	59	-0.83	91	-0.83
28	-0.98	60	-0.98	92	-0.98
29	-0.98	61	-0.98	93	-0.98
30	-0.83	62	-0.83	94	-0.83
31	-0.56	63	-0.56	95	-0.56
32	-0.20	64	-0.20	96	-0.20

EACH CHANNEL = 22.50 DEGREES

SIN 7X = 7TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.23	33	0.73	65	-0.96
2	0.63	34	0.35	66	-0.99
3	0.91	35	-0.10	67	-0.81
4	1.00	36	-0.53	68	-0.47
5	0.88	37	-0.85	69	-0.03
6	0.58	38	-1.00	70	0.41
7	0.16	39	-0.94	71	0.77
8	-0.29	40	-0.68	72	0.97
9	-0.68	41	-0.29	73	0.97
10	-0.94	42	0.16	74	0.77
11	-1.00	43	0.58	75	0.41
12	-0.85	44	0.88	76	-0.03
13	-0.53	45	1.00	77	-0.47
14	-0.10	46	0.91	78	-0.81
15	0.35	47	0.63	79	-0.99
16	0.73	48	0.23	80	-0.96
17	0.96	49	-0.23	81	-0.73
18	0.99	50	-0.63	82	-0.35
19	0.81	51	-0.91	83	0.10
20	0.47	52	-1.00	84	0.53
21	0.03	53	-0.88	85	0.85
22	-0.41	54	-0.58	86	1.00
23	-0.77	55	-0.16	87	0.94
24	-0.97	56	0.29	88	0.68
25	-0.97	57	0.68	89	0.29
26	-0.77	58	0.94	90	-0.16
27	-0.41	59	1.00	91	-0.58
28	0.03	60	0.85	92	-0.88
29	0.47	61	0.53	93	-1.00
30	0.81	62	0.10	94	-0.91
31	0.99	63	-0.35	95	-0.63
32	0.96	64	-0.73	96	-0.23

EACH CHANNEL = 26.25 DEGREES

SIN 8X = 8TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.26	33	-0.97	65	0.71
2	0.71	34	-0.97	66	0.26
3	0.97	35	-0.71	67	-0.26
4	0.97	36	-0.26	68	-0.71
5	0.71	37	0.26	69	-0.97
6	0.26	38	0.71	70	-0.97
7	-0.26	39	0.97	71	-0.71
8	-0.71	40	0.97	72	-0.26
9	-0.97	41	0.71	73	0.26
10	-0.97	42	0.26	74	0.71
11	-0.71	43	-0.26	75	0.97
12	-0.26	44	-0.71	76	0.97
13	0.26	45	-0.97	77	0.71
14	0.71	46	-0.97	78	0.26
15	0.97	47	-0.71	79	-0.26
16	0.97	48	-0.26	80	-0.71
17	0.71	49	0.26	81	-0.97
18	0.26	50	0.71	82	-0.97
19	-0.26	51	0.97	83	-0.71
20	-0.71	52	0.97	84	-0.26
21	-0.97	53	0.71	85	0.26
22	-0.97	54	0.26	86	0.71
23	-0.71	55	-0.26	87	0.97
24	-0.26	56	-0.71	88	0.97
25	0.26	57	-0.97	89	0.71
26	0.71	58	-0.97	90	0.26
27	0.97	59	-0.71	91	-0.26
28	0.97	60	-0.26	92	-0.71
29	0.71	61	0.26	93	-0.97
30	0.26	62	0.71	94	-0.97
31	-0.26	63	0.97	95	-0.71
32	-0.71	64	0.97	96	-0.26

EACH CHANNEL = 30.00 DEGREES

SIN 9X = 9TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.29	33	0.29	65	0.29
2	0.77	34	0.77	66	0.77
3	1.00	35	1.00	67	1.00
4	0.88	36	0.88	68	0.88
5	0.47	37	0.47	69	0.47
6	-0.10	38	-0.10	70	-0.10
7	-0.63	39	-0.63	71	-0.63
8	-0.96	40	-0.96	72	-0.96
9	-0.96	41	-0.96	73	-0.96
10	-0.63	42	-0.63	74	-0.63
11	-0.10	43	-0.10	75	-0.10
12	0.47	44	0.47	76	0.47
13	0.88	45	0.88	77	0.88
14	1.00	46	1.00	78	1.00
15	0.77	47	0.77	79	0.77
16	0.29	48	0.29	80	0.29
17	-0.29	49	-0.29	81	-0.29
18	-0.77	50	-0.77	82	-0.77
19	-1.00	51	-1.00	83	-1.00
20	-0.88	52	-0.88	84	-0.88
21	-0.47	53	-0.47	85	-0.47
22	0.10	54	0.10	86	0.10
23	0.63	55	0.63	87	0.63
24	0.96	56	0.96	88	0.96
25	0.96	57	0.96	89	0.96
26	0.63	58	0.63	90	0.63
27	0.10	59	0.10	91	0.10
28	-0.47	60	-0.47	92	-0.47
29	-0.88	61	-0.88	93	-0.88
30	-1.00	62	-1.00	94	-1.00
31	-0.77	63	-0.77	95	-0.77
32	-0.29	64	-0.29	96	-0.29

EACH CHANNEL = 33.75 DEGREES

SIN 10X = 10TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.32	33	0.66	65	-0.98
2	0.83	34	0.07	66	-0.90
3	1.00	35	-0.56	67	-0.44
4	0.75	36	-0.95	68	0.20
5	0.20	37	-0.95	69	0.75
6	-0.44	38	-0.56	70	1.00
7	-0.90	39	0.07	71	0.83
8	-0.98	40	0.66	72	0.32
9	-0.66	41	0.98	73	-0.32
10	-0.07	42	0.90	74	-0.83
11	0.56	43	0.44	75	-1.00
12	0.95	44	-0.20	76	-0.75
13	0.95	45	-0.75	77	-0.20
14	0.56	46	-1.00	78	0.44
15	-0.07	47	-0.83	79	0.90
16	-0.66	48	-0.32	80	0.98
17	-0.98	49	0.32	81	0.66
18	-0.90	50	0.83	82	0.07
19	-0.44	51	1.00	83	-0.56
20	0.20	52	0.75	84	-0.95
21	0.75	53	0.20	85	-0.95
22	1.00	54	-0.44	86	-0.56
23	0.83	55	-0.90	87	0.07
24	0.32	56	-0.98	88	0.66
25	-0.32	57	-0.66	89	0.98
26	-0.83	58	-0.07	90	0.90
27	-1.00	59	0.56	91	0.44
28	-0.75	60	0.95	92	-0.20
29	-0.20	61	0.95	93	-0.75
30	0.44	62	0.56	94	-1.00
31	0.90	63	-0.07	95	-0.83
32	0.98	64	-0.66	96	-0.32

EACH CHANNEL = 37.50 DEGREES

SIN 11X = 11TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.35	33	-0.99	65	0.63
2	0.88	34	-0.85	66	-0.03
3	0.97	35	-0.29	67	-0.68
4	0.58	36	0.41	68	-1.00
5	-0.10	37	0.91	69	-0.81
6	-0.73	38	0.96	70	-0.23
7	-1.00	39	0.53	71	0.47
8	-0.77	40	-0.16	72	0.94
9	-0.16	41	-0.77	73	0.94
10	0.53	42	-1.00	74	0.47
11	0.96	43	-0.73	75	-0.23
12	0.91	44	-0.10	76	-0.81
13	0.41	45	0.58	77	-1.00
14	-0.29	46	0.97	78	-0.68
15	-0.85	47	0.88	79	-0.03
16	-0.99	48	0.35	80	0.63
17	-0.63	49	-0.35	81	0.99
18	0.03	50	-0.88	82	0.85
19	0.68	51	-0.97	83	0.29
20	1.00	52	-0.58	84	-0.41
21	0.81	53	0.10	85	-0.91
22	0.23	54	0.73	86	-0.96
23	-0.47	55	1.00	87	-0.53
24	-0.94	56	0.77	88	0.16
25	-0.94	57	0.16	89	0.77
26	-0.47	58	-0.53	90	1.00
27	0.23	59	-0.96	91	0.73
28	0.81	60	-0.91	92	0.10
29	1.00	61	-0.41	93	-0.58
30	0.68	62	0.29	94	-0.97
31	0.03	63	0.85	95	-0.88
32	-0.63	64	0.99	96	-0.35

EACH CHANNEL = 41.25 DEGREES

SIN 12X = 12TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.38	33	0.38	65	0.38
2	0.92	34	0.92	66	0.92
3	0.92	35	0.92	67	0.92
4	0.38	36	0.38	68	0.38
5	-0.38	37	-0.38	69	-0.38
6	-0.92	38	-0.92	70	-0.92
7	-0.92	39	-0.92	71	-0.92
8	-0.38	40	-0.38	72	-0.38
9	0.38	41	0.38	73	0.38
10	0.92	42	0.92	74	0.92
11	0.92	43	0.92	75	0.92
12	0.38	44	0.38	76	0.38
13	-0.38	45	-0.38	77	-0.38
14	-0.92	46	-0.92	78	-0.92
15	-0.92	47	-0.92	79	-0.92
16	-0.38	48	-0.38	80	-0.38
17	0.38	49	0.38	81	0.38
18	0.92	50	0.92	82	0.92
19	0.92	51	0.92	83	0.92
20	0.38	52	0.38	84	0.38
21	-0.38	53	-0.38	85	-0.38
22	-0.92	54	-0.92	86	-0.92
23	-0.92	55	-0.92	87	-0.92
24	-0.38	56	-0.38	88	-0.38
25	0.38	57	0.38	89	0.38
26	0.92	58	0.92	90	0.92
27	0.92	59	0.92	91	0.92
28	0.38	60	0.38	92	0.38
29	-0.38	61	-0.38	93	-0.38
30	-0.92	62	-0.92	94	-0.92
31	-0.92	63	-0.92	95	-0.92
32	-0.38	64	-0.38	96	-0.38

EACH CHANNEL = 45.00 DEGREES

SIN 13X = 13TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.41	33	0.58	65	-1.00
2	0.96	34	-0.23	66	-0.73
3	0.85	35	-0.88	67	0.03
4	0.16	36	-0.94	68	0.77
5	-0.63	37	-0.35	69	0.99
6	-1.00	38	0.47	70	0.53
7	-0.68	39	0.97	71	-0.29
8	0.10	40	0.81	72	-0.91
9	0.81	41	0.10	73	-0.91
10	0.97	42	-0.68	74	-0.29
11	0.47	43	-1.00	75	0.53
12	-0.35	44	-0.63	76	0.99
13	-0.94	45	0.16	77	0.77
14	-0.88	46	0.85	78	0.03
15	-0.23	47	0.96	79	-0.73
16	0.58	48	0.41	80	-1.00
17	1.00	49	-0.41	81	-0.58
18	0.73	50	-0.96	82	0.23
19	-0.03	51	-0.85	83	0.88
20	-0.77	52	-0.16	84	0.94
21	-0.99	53	0.63	85	0.35
22	-0.53	54	1.00	86	-0.47
23	0.29	55	0.68	87	-0.97
24	0.91	56	-0.10	88	-0.81
25	0.91	57	-0.81	89	-0.10
26	0.29	58	-0.97	90	0.68
27	-0.53	59	-0.47	91	1.00
28	-0.99	60	0.35	92	0.63
29	-0.77	61	0.94	93	-0.16
30	-0.03	62	0.88	94	-0.85
31	0.73	63	0.23	95	-0.96
32	1.00	64	-0.58	96	-0.41

EACH CHANNEL = 48.75 DEGREES

SIN 14X = 14TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.44	33	-1.00	65	0.56
2	0.98	34	-0.66	66	-0.32
3	0.75	35	0.20	67	-0.95
4	-0.07	36	0.90	68	-0.83
5	-0.83	37	0.90	69	-0.07
6	-0.95	38	0.20	70	0.75
7	-0.32	39	-0.66	71	0.98
8	0.56	40	-1.00	72	0.44
9	1.00	41	-0.56	73	-0.44
10	0.66	42	0.32	74	-0.98
11	-0.20	43	0.95	75	-0.75
12	-0.90	44	0.83	76	0.07
13	-0.90	45	0.07	77	0.83
14	-0.20	46	-0.75	78	0.95
15	0.66	47	-0.98	79	0.32
16	1.00	48	-0.44	80	-0.56
17	0.56	49	0.44	81	-1.00
18	-0.32	50	0.98	82	-0.66
19	-0.95	51	0.75	83	0.20
20	-0.83	52	-0.07	84	0.90
21	-0.07	53	-0.83	85	0.90
22	0.75	54	-0.95	86	0.20
23	0.98	55	-0.32	87	-0.66
24	0.44	56	0.56	88	-1.00
25	-0.44	57	1.00	89	-0.56
26	-0.98	58	0.66	90	0.32
27	-0.75	59	-0.20	91	0.95
28	0.07	60	-0.90	92	0.83
29	0.83	61	-0.90	93	0.07
30	0.95	62	-0.20	94	-0.75
31	0.32	63	0.66	95	-0.98
32	-0.56	64	1.00	96	-0.44

EACH CHANNEL = 52.50 DEGREES

SIN 16X = 16TH HARMONIC

CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE	CHANNEL	AMPLITUDE
1	0.50	33	0.50	65	-1.00
2	1.00	34	-0.50	66	-0.50
3	0.50	35	-1.00	67	0.50
4	-0.50	36	-0.50	68	1.00
5	-1.00	37	0.50	69	0.50
6	-0.50	38	1.00	70	-0.50
7	0.50	39	0.50	71	-1.00
8	1.00	40	-0.50	72	-0.50
9	0.50	41	-1.00	73	0.50
10	-0.50	42	-0.50	74	1.00
11	-1.00	43	0.50	75	0.50
12	-0.50	44	1.00	76	-0.50
13	0.50	45	0.50	77	-1.00
14	1.00	46	-0.50	78	-0.50
15	0.50	47	-1.00	79	0.50
16	-0.50	48	-0.50	80	1.00
17	-1.00	49	0.50	81	0.50
18	-0.50	50	1.00	82	-0.50
19	0.50	51	0.50	83	-1.00
20	1.00	52	-0.50	84	-0.50
21	0.50	53	-1.00	85	0.50
22	-0.50	54	-0.50	86	1.00
23	-1.00	55	0.50	87	0.50
24	-0.50	56	1.00	88	-0.50
25	0.50	57	0.50	89	-1.00
26	1.00	58	-0.50	90	-0.50
27	0.50	59	-1.00	91	0.50
28	-0.50	60	-0.50	92	1.00
29	-1.00	61	0.50	93	0.50
30	-0.50	62	1.00	94	-0.50
31	0.50	63	0.50	95	-1.00
32	1.00	64	-0.50	96	-0.50

EACH CHANNEL = 60.00 DEGREES

APPENDIX F

Power Supply Schematics

All power supplies for the AWG are regulated. Figure 25 shows that ± 10 volt power supply used mainly for the scaler and matrix. Extreme regulation was not called for and hence emitter-follower type regulators were used, being quite capable of supplying the heavy current drains of these circuits. The 22 volt power supply, Figure 26, used mainly in the frequency multiplication circuits requires quite good regulation. A series regulator with a differential comparitor error-sensing element provides the regulation. A constant current source improves the operation of the differential amplifier. Figure 27 shows the potentiometer power supply whose operation is similar to that of the 22 volt supply except that two series transistors are provided to establish a "floating" dc return. The regulation occurs between +1 and +6 volts rather than with respect to ground.

APPENDIX G

Taped Examples

Accompanying this thesis are taped examples of waveforms generated by the AWG. The recording was made on an Ampex Model 354 2-channel 1/4 inch tape recorder. They were made at a tape speed of 7 1/2 i.p.s. on channel A. Table IV lists the waveforms found on the tape.

TABLE IV

Taped Examples

Number	Spectrum	Frequency
1	sin x + sin 2x + 1/2 sin 3x + 3/5 sin 4x + 1/10 sin 5x + 3/100 6x (Trombone)	256 cps
2	.3 sin x + sin 2x + .2 sin 3x + .35 sin 4x + .12 sin 5x + .18 sin 6x + .25 sin 7x + .3 sin 8x + .33 sin 9x + .25 sin 10x + .08 sin 11x (Vowel "A")	256 cps
3	sin x + .43 sin 2x + .23 sin 3x + .15 sin 4x + .12 sin 5x + .2 sin 7x + .35 sin 8x + .9 sin 9x + .88 sin 10x + .18 sin 11x + .2 sin 15x + .2 sin 16x (Vowel "E")	256 cps
4	.85 sin (x + 120°) + .75 sin (2x + 200°) + .6 sin 3x + .5 sin (4x + 175°) + .4 sin (5x + 300°) + .33 sin (6x + 90°) + .23 sin (7x + 200°) + .18 sin 8x (Trumpet)	932 cps

Number		
5	$\sin x + .11 \sin 2x + .35 \sin 3x + .04 \sin 4x + .32 \sin 5x + .11 \sin 6x + .32 \sin 7x + .2 \sin 8x + .17 \sin 9x + .08 \sin 10x + .06 \sin 11x$ (Clarinet)	256 cps
6	$\sin x + \sin 16x$	sweep 260 cps --20 cps
7	$\sin x + 1/3\,[\sin(13x + 146°) + \sin(14x + 158°) + \sin(15x + 169°)]$	200 cps
8	$\sin x + .85 \sin 3x + \sin 4x + .85 \sin(6x + 293°) + .5 \sin(7x + 131°)$	340 cps
9	$\sum_{n=1}^{16} \sin(nx)$	128 cps
10	$\sum_{n=1}^{16} \frac{n}{10} \sin(nx)$	100 cps
11	$\sum_{n=1}^{8} \sin[(2n-1)x]$	100, 50, and 200 cps
12	$\sin x + \sin(7x + 289°) + 1/2 \sin(12x + 180°)$	325 cps

REFERENCES

James W. Beauchamp, "A Statement of Progress on the Research Investigation, 'Generation and Creation of New Electronic Sounds'", Technical Report No. 7, Univ. Ill. School of Music Experimental Music Studio, Aug., 1963.

James W. Beauchamp, "A Report on the Magnovox Sponsored Research Investigation 'The Development of New Electronic Systems for Generating Musical Sound'", Technical Report No. 10, Univ. Ill. School of Music Experimental Music Studio, Aug., 1964.

Leo L. Beranek, Acoustics (McGraw-Hill Book Company., Inc., New York, 1954), pp. 397, 398.

Melville Clark, Jr., David Luce, Robert Abrams, Howard Schlossberg, and James Rome, J. Aud. Eng. Soc., 11, No. 1, p. 45 (1963).

Melville Clark, Jr., and Paul Miller, J. Aud. Eng. Soc., 12, No. 1, p. 28 (1964).

Harvey Fletcher and W. W. Munson, J. Acous. Soc. Amer., 5, No. 2, p. 82 (1933).

G. E. Transistor Manual, Seventh Ed., 1964, pp. 316-347.

M. G. Hartley, An Introduction to Electronic Analogue Computers, New York: John Wiley and Sons Inc., 1962 (Chapter 3).

L. A. Hiller, "Musical Composition with a High Speed Digital Computer", J. Aud. Eng. Soc., 6, No. 3, p. 154 (1958).

D. A. Luce, "Physical Correlates of Nonpercussive Musical Instrument Tones", MIT Ph.D. Thesis, 1963.

REFERENCES
(cont.)

H. F. Olson, Elements of Acoustical Engineering, New York: D. van Nostrand Co., Inc., 1942, p. 332.

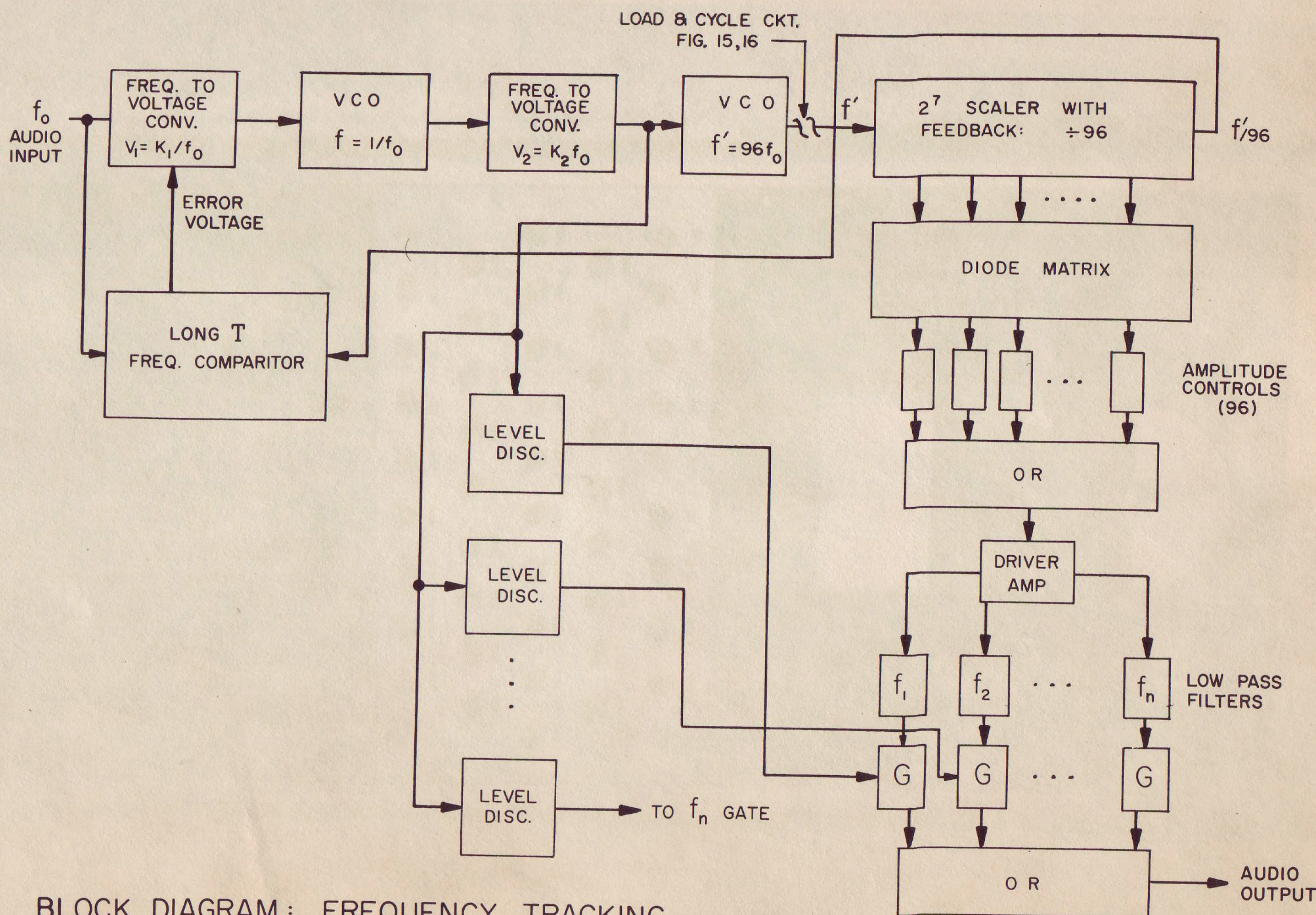
H. F. Olson and H. Belar, J. Acous. Soc. Am., 27, No. 3, May 1955, pp. 595-612.

RCA Industrial Transistor Data Sheet for 2N3053/40053, 2N3054, 2N3055 (12-1963).

W. H. Voelker, "Transistor Circuit Converts Voltage to Regulated Frequency", Electronics, Nov. 16, 1964, Vol. 37, No. 29, p. 73.

FIGURE CAPTIONS

GRAPH NO. G65-28



BLOCK DIAGRAM: FREQUENCY TRACKING WAVEFORM SYNTHESIZER

FIGURE 1

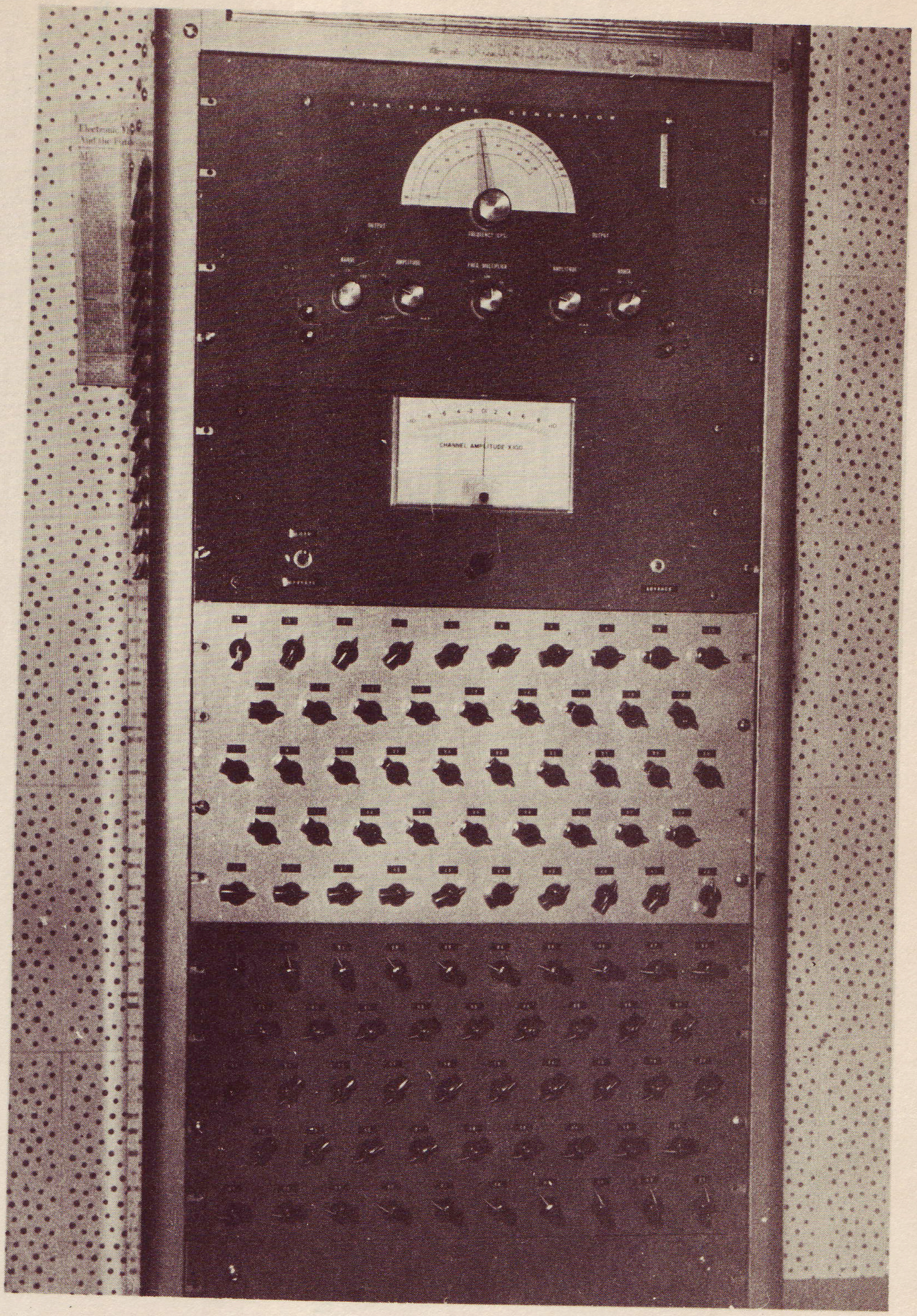
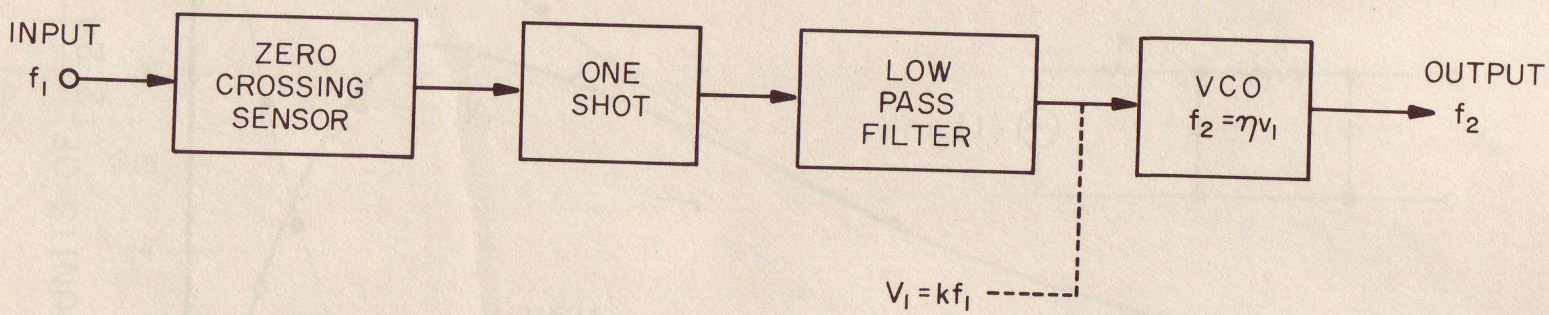


FIGURE 2



BLOCK DIAGRAM

PULSE AVERAGING FREQUENCY MULTIPLIER

FIGURE 3

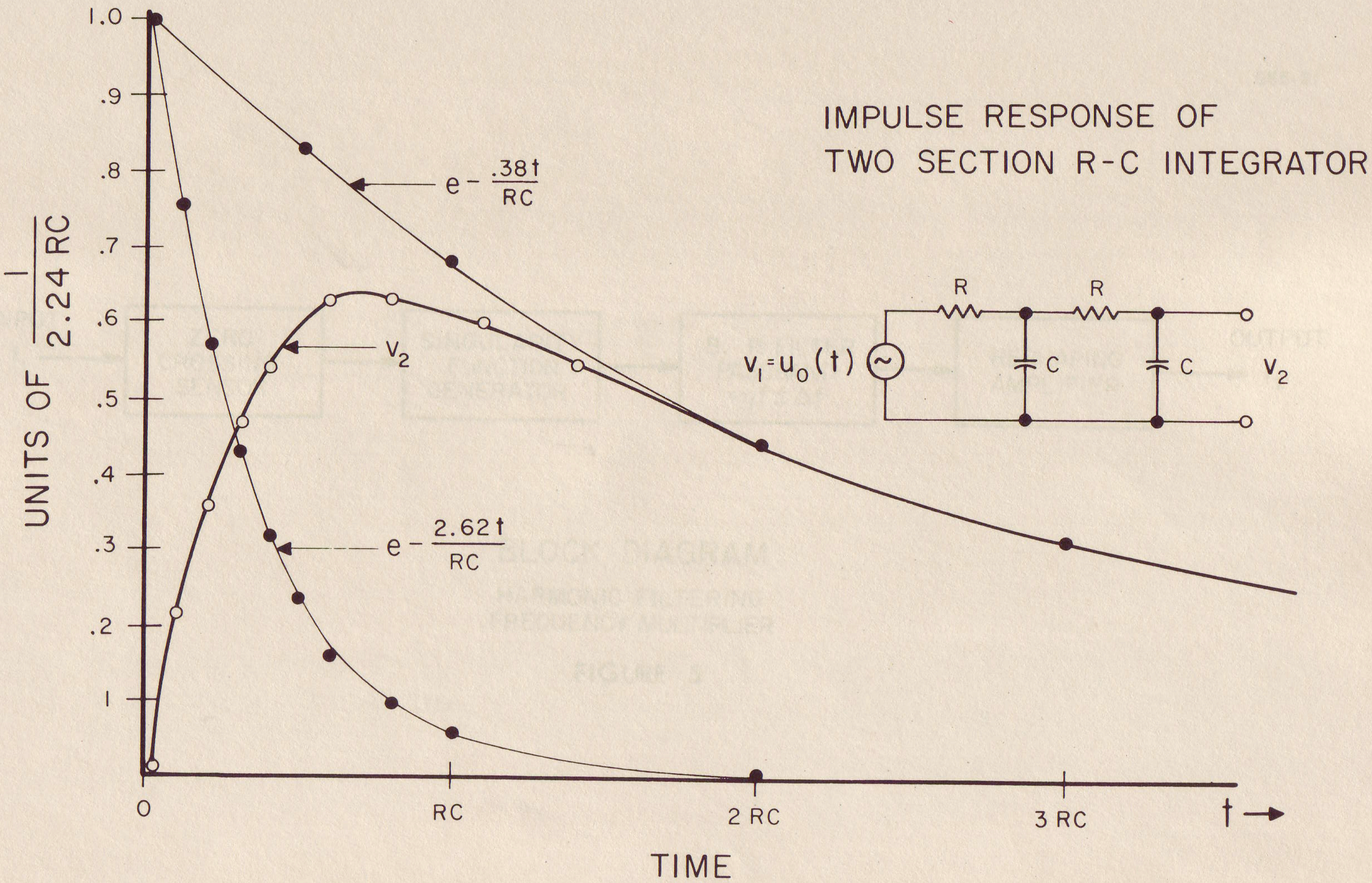
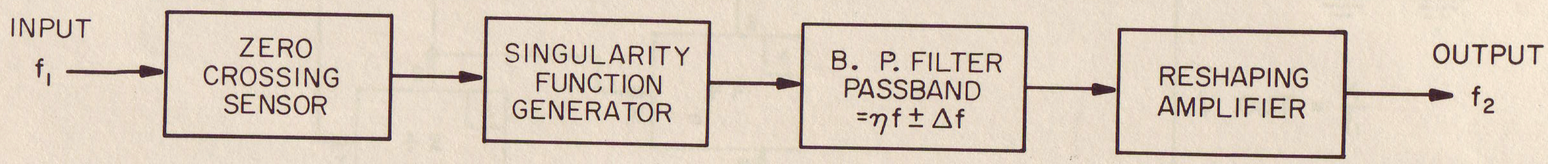

IMPULSE RESPONSE OF
TWO SECTION R-C INTEGRATOR
UNITS OF $\frac{1}{2.24\,RC}$
1.0
.9
.8
.7
.6
.5
.4
.3
.2
1
0
RC
2 RC
3 RC
t →
TIME
$e^{-\frac{.38t}{RC}}$
v_2
$e^{-\frac{2.62t}{RC}}$
$v_1 = u_0(t)$
R
R
C
C
v_2

FIGURE 4

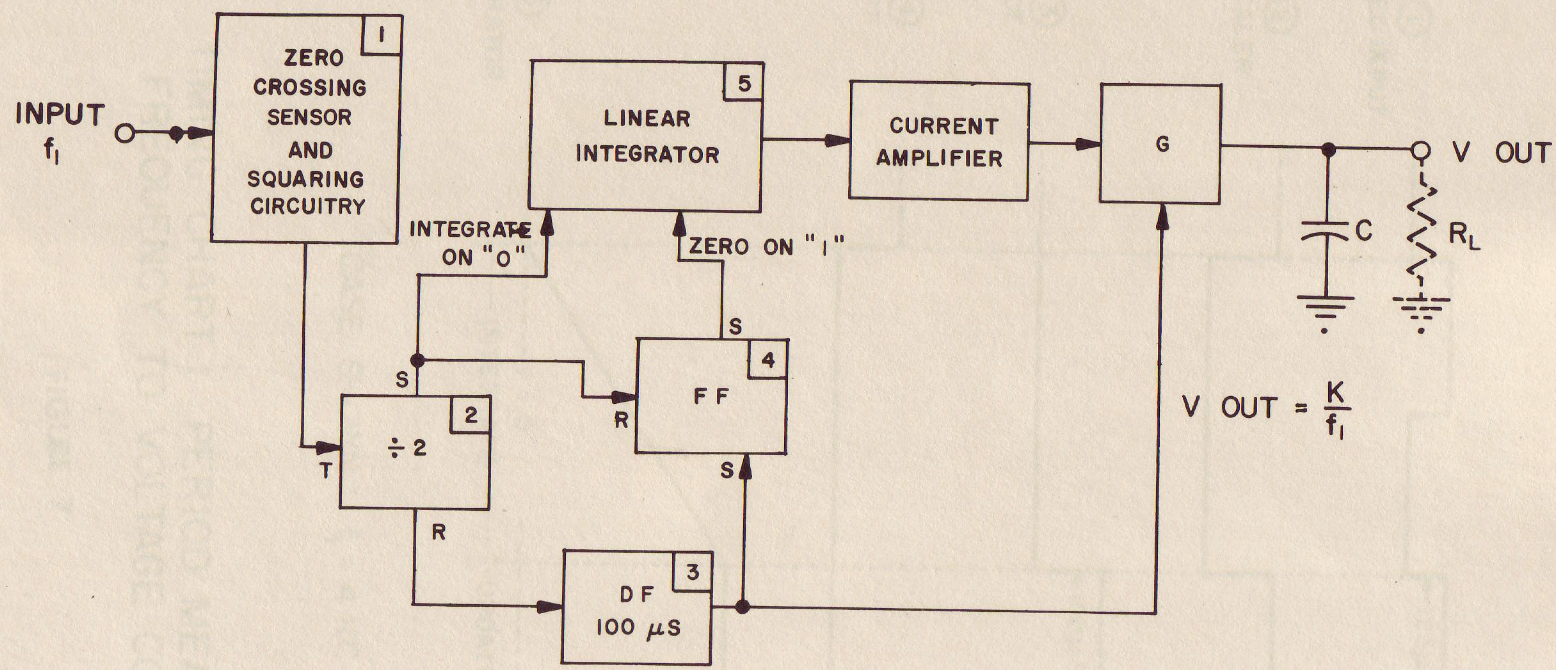
G65-21



BLOCK DIAGRAM

HARMONIC FILTERING
FREQUENCY MULTIPLIER

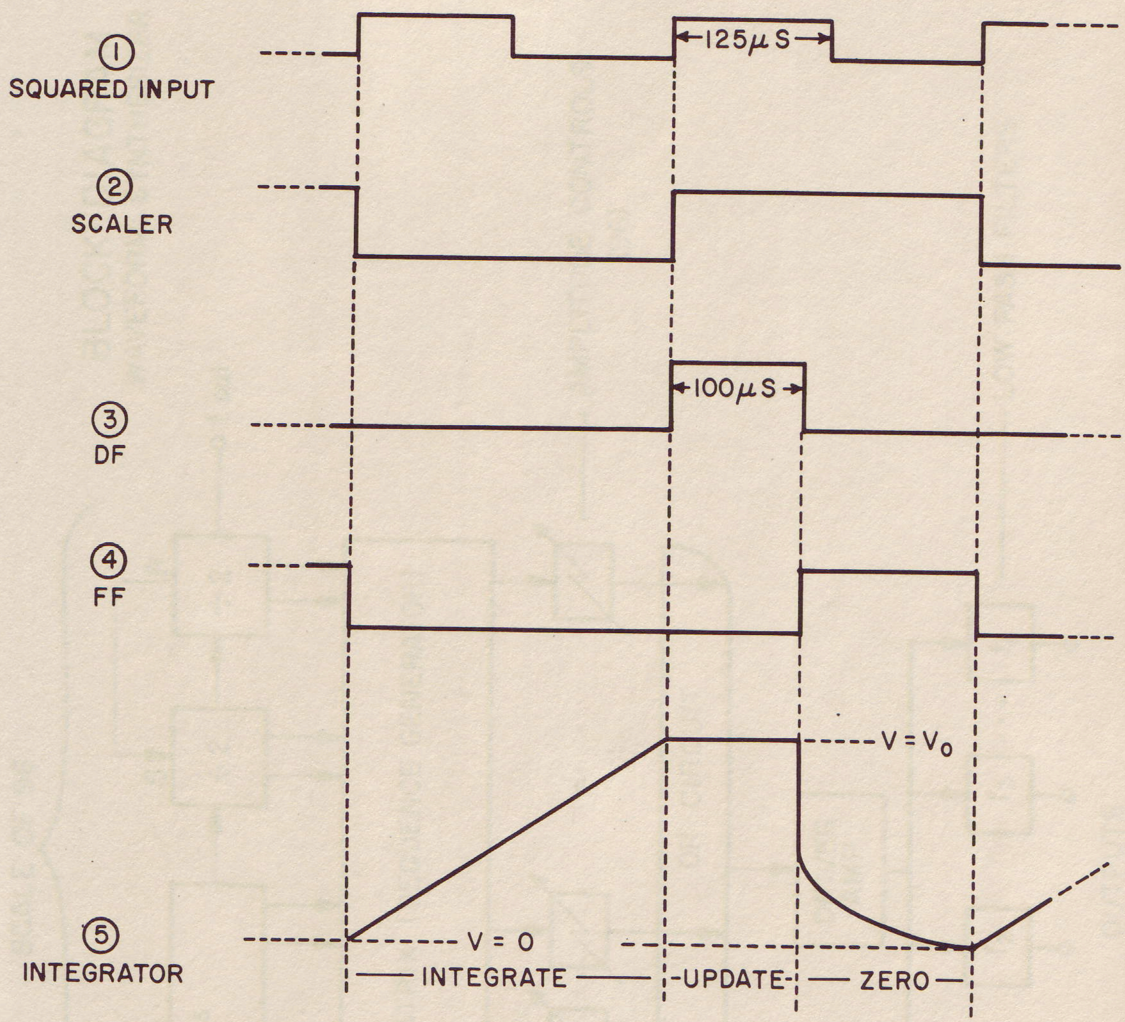
FIGURE 5



NOTE: NUMBERED BLOCKS CORRESPOND TO NUMBERS ON TIMING CHART

BLOCK DIAGRAM,
PERIOD MEASUREMENT
FREQUENCY TO VOLTAGE
CONVERTER

FIGURE 6



CASE SHOWN : f_1 = 4 KC

TIMING CHART : PERIOD MEASUREMENT
FREQUENCY TO VOLTAGE CONVERTER

FIGURE 7

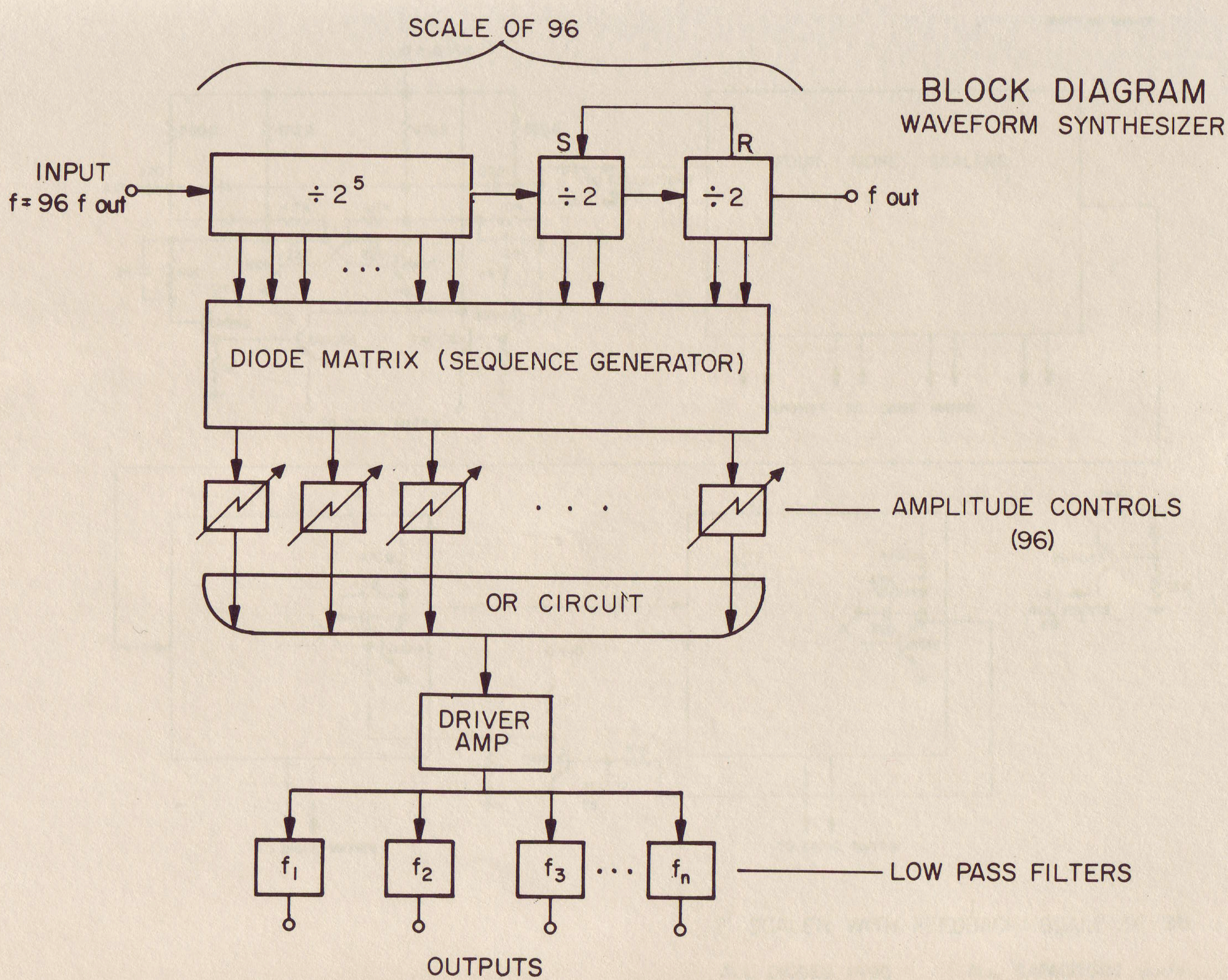
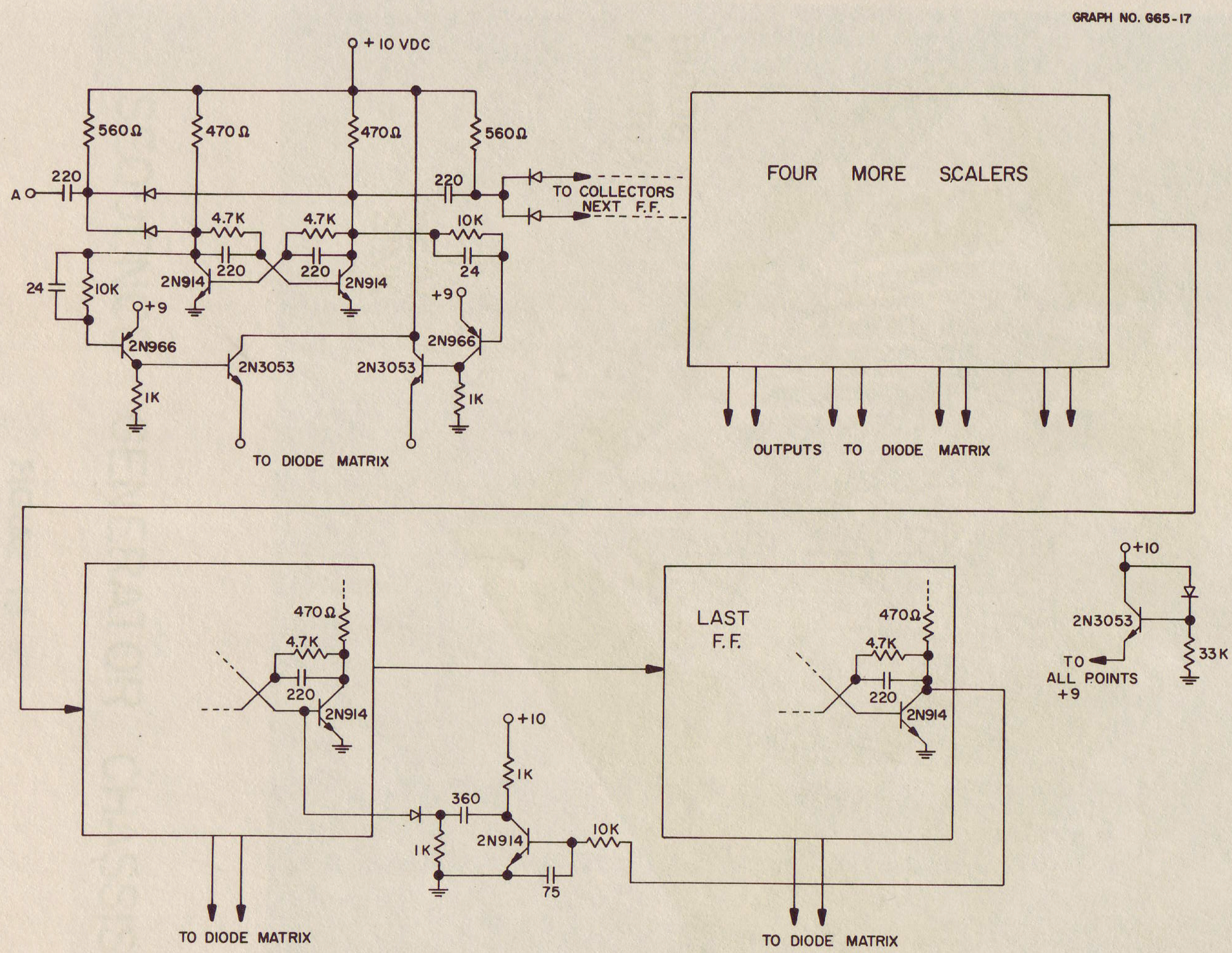

BLOCK DIAGRAM
WAVEFORM SYNTHESIZER
SCALE OF 96
INPUT
f = 96 f out
÷ 2^5
S
÷ 2
R
÷ 2
f out
DIODE MATRIX (SEQUENCE GENERATOR)
AMPLITUDE CONTROLS
(96)
OR CIRCUIT
DRIVER
AMP
f1
f2
f3
fn
LOW PASS FILTERS
OUTPUTS

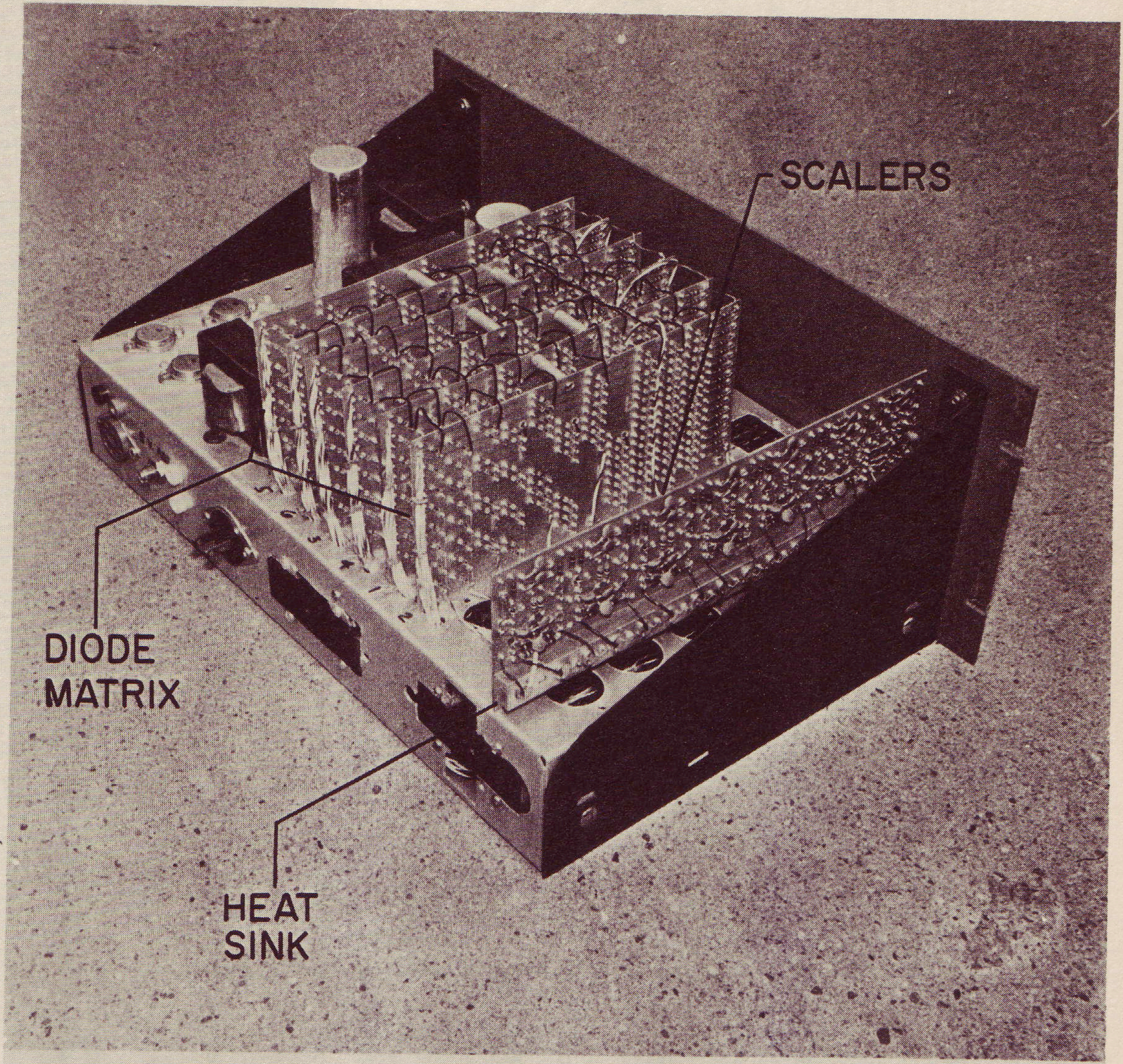
FIGURE 8



2^7 SCALER WITH FEEDBACK: SCALE OF 96

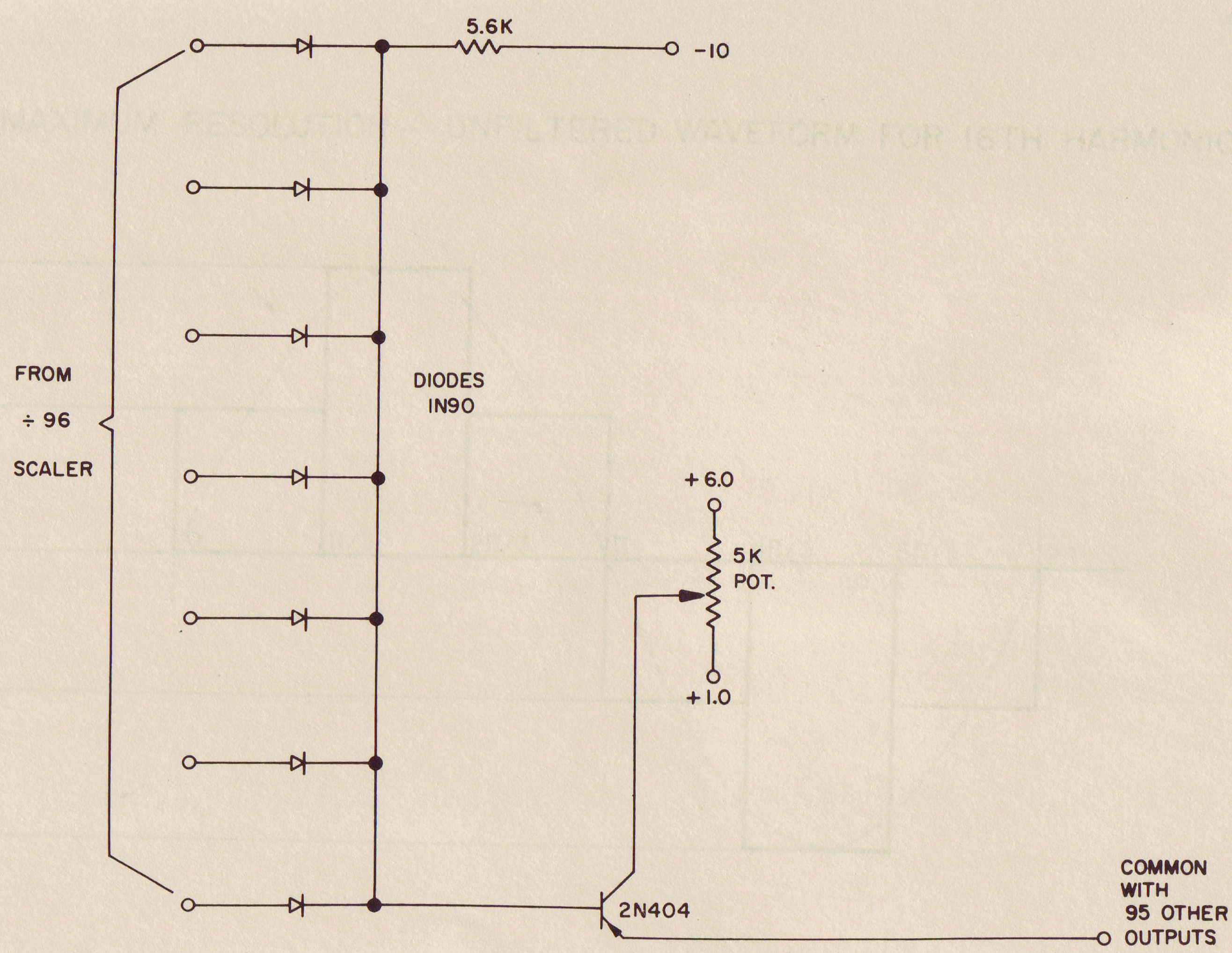
ALL DIODES IN90 ALL CAPACITORS $\mu\mu$fd

FIGURE 9



SEQUENCE GENERATOR CHASSIS

FIGURE 10



DIODE MATRIX, MEMORY, OR-GATE, SCHEMATIC — ONE ELEMENT

FIGURE 11

MAXIMUM RESOLUTION --- UNFILTERED WAVEFORM FOR 16TH HARMONIC

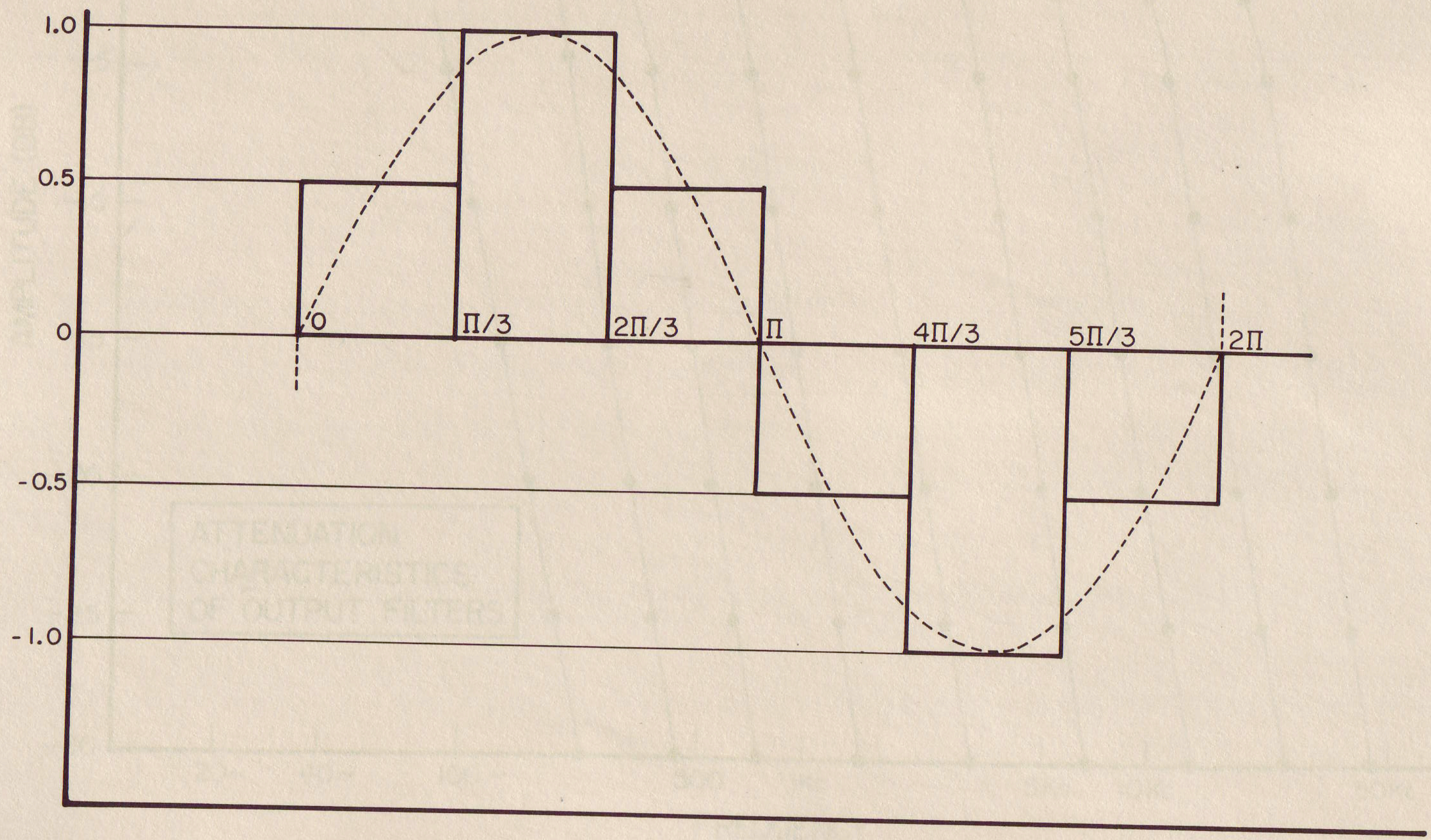


FIGURE 12

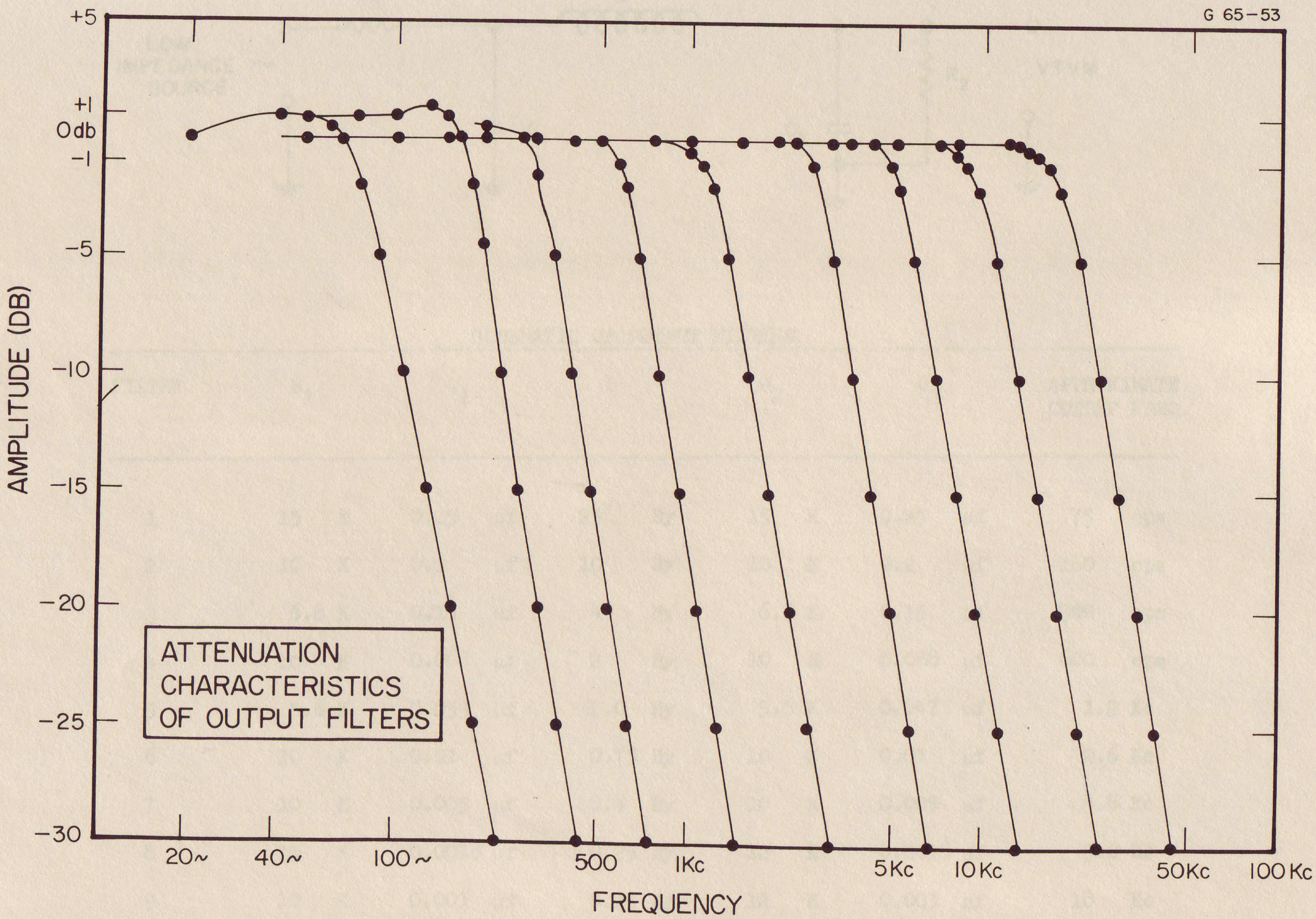
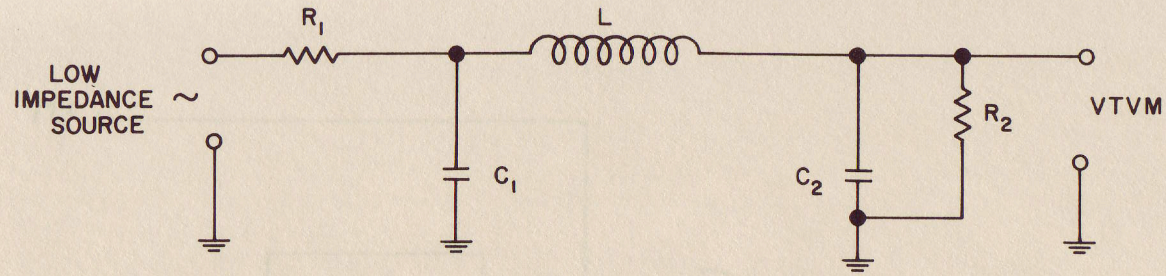

G 65-53
ATTENUATION
CHARACTERISTICS
OF OUTPUT FILTERS
AMPLITUDE (DB)
+5
+1
0db
-1
-5
-10
-15
-20
-25
-30
20~
40~
100~
500
1Kc
5Kc
10Kc
50Kc
100Kc
FREQUENCY

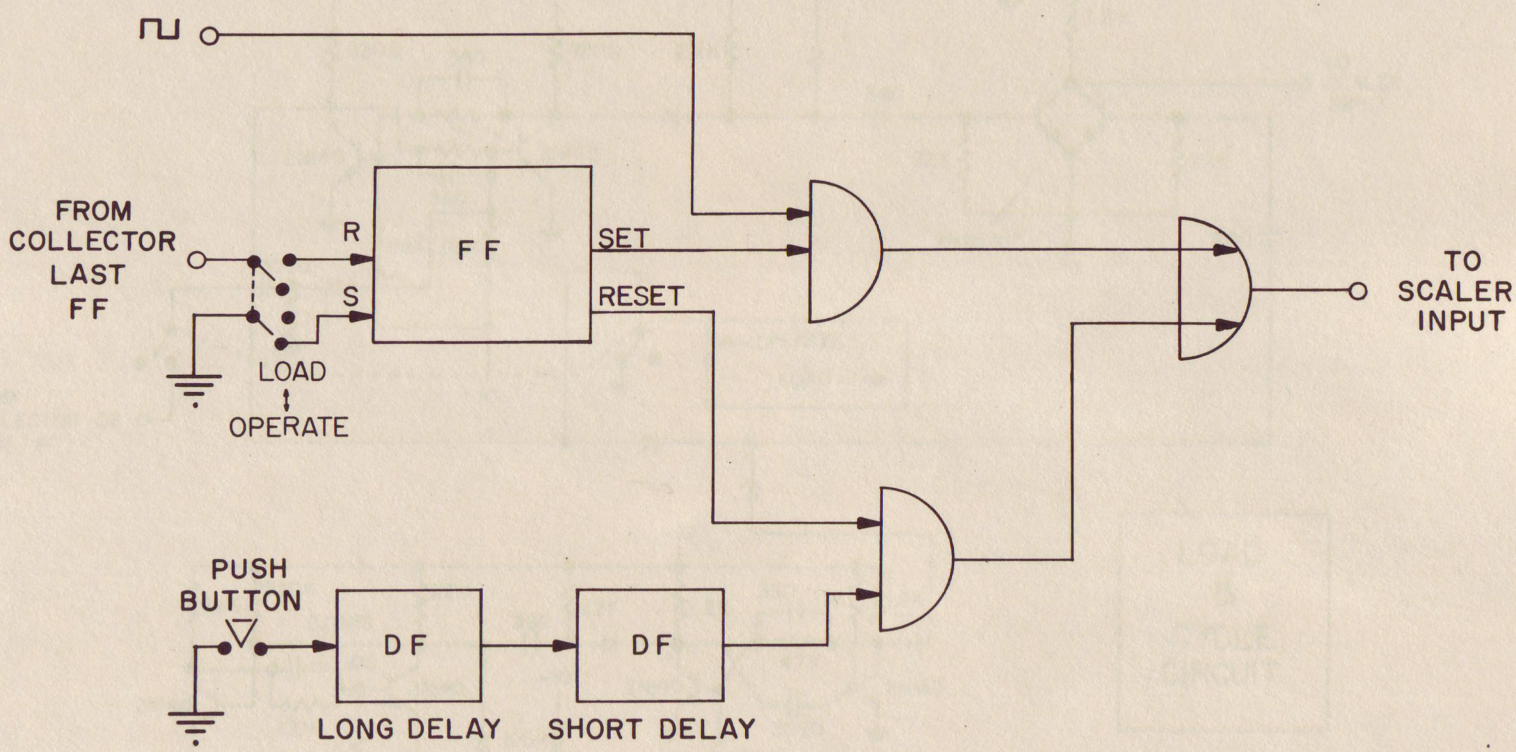
FIGURE 13



SCHEMATIC OF OUTPUT FILTERS

FILTER	R_1	C_1	L	R_2	C_2	APPROXIMATE CUTOFF FREQ.
1	15 K	0.25 µf	25 Hy	15 K	0.25 µf	75 cps
2	10 K	0.2 µf	10 Hy	10 K	0.2 µf	160 cps
3	6.8 K	0.15 µf	4 Hy	6.8 K	0.15 µf	300 cps
4	10 K	0.068 µf	2 Hy	10 K	0.068 µf	600 cps
5	5.6 K	0.033 µf	1.0 Hy	5.6 K	0.047 µf	1.2 Kc
6	10 K	0.01 µf	0.75 Hy	10 K	0.01 µf	2.6 Kc
7	10 K	0.005 µf	0.4 Hy	10 K	0.005 µf	4.8 Kc
8	10 K	0.0018 µf	0.25 Hy	12 K	0.0018 µf	9.0 Kc
9	10 K	0.001 µf	0.15 Hy	12 K	0.001 µf	18 Kc

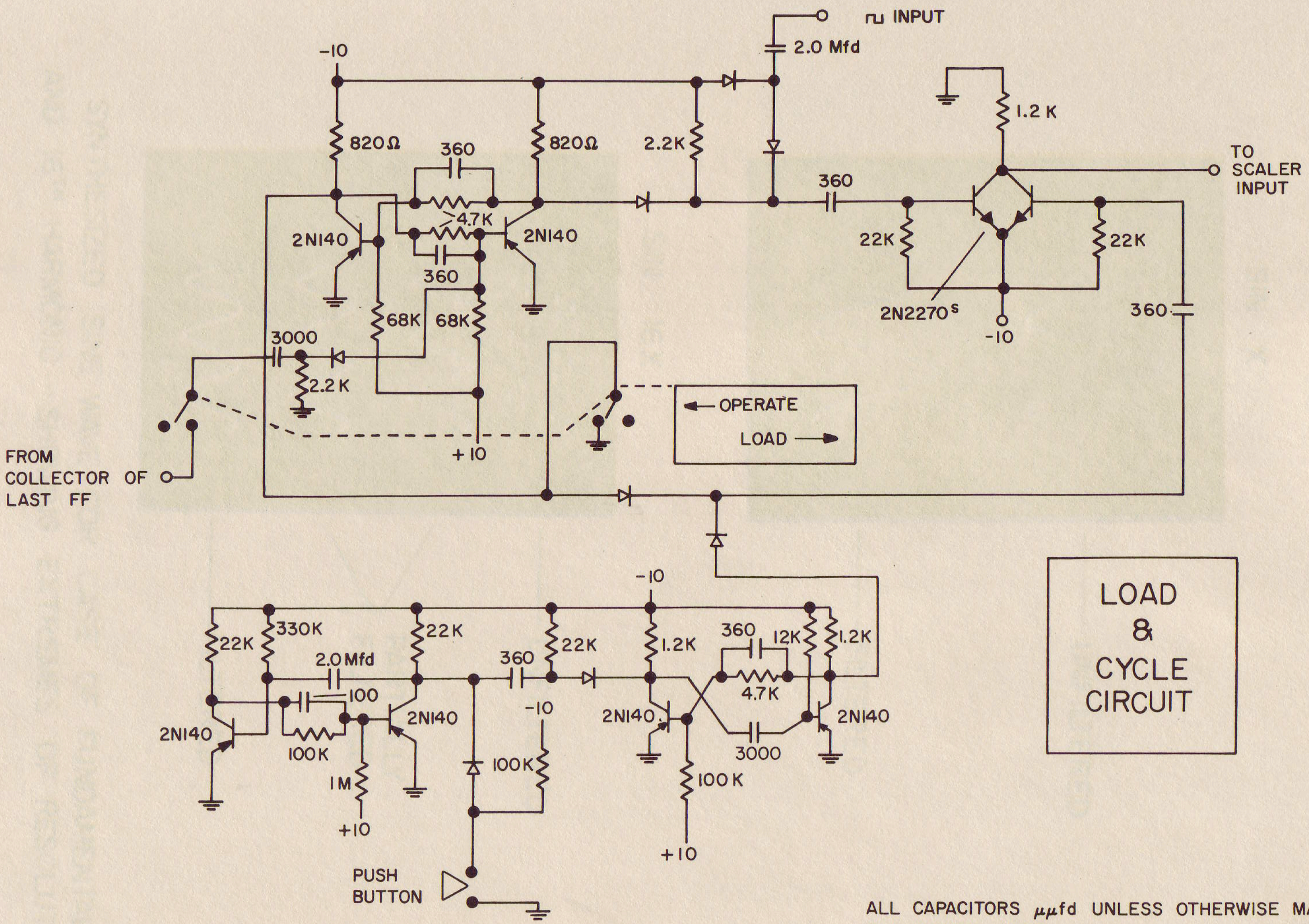
FIGURE 14



BLOCK DIAGRAM
LOAD AND CYCLE CIRCUIT

FIGURE 15

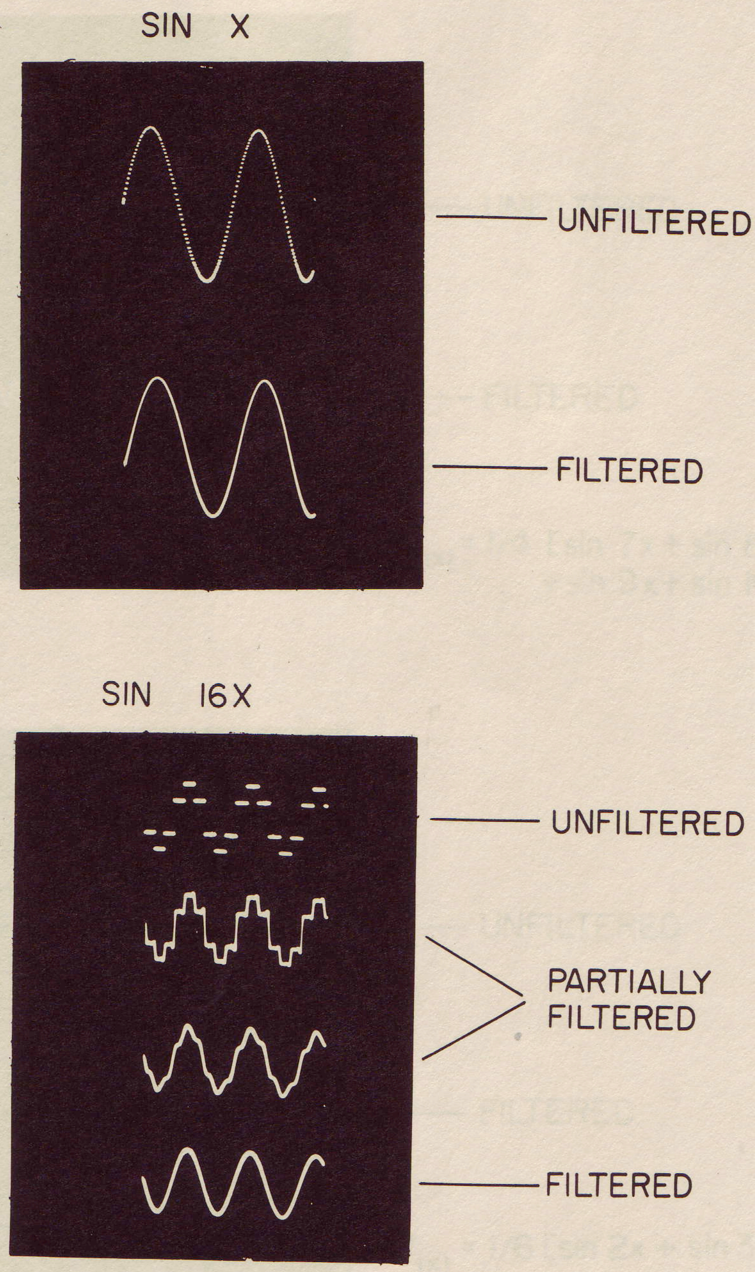
GRAPH NO. G65-18



ALL CAPACITORS μμfd UNLESS OTHERWISE MARKED
ALL DIODES IN90

FIGURE 16

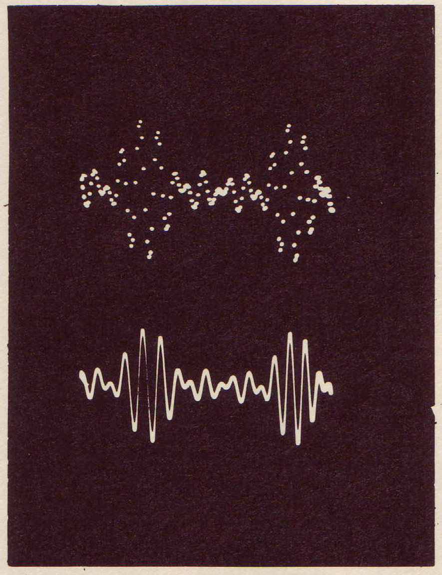
GRAPH NO. G65 - 26



SYNTHESIZED SINE WAVE FOR CASE OF FUNDAMENTAL AND 16TH HARMONIC ··· SHOWING EXTREMES OF RESOLUTION

FIGURE 17

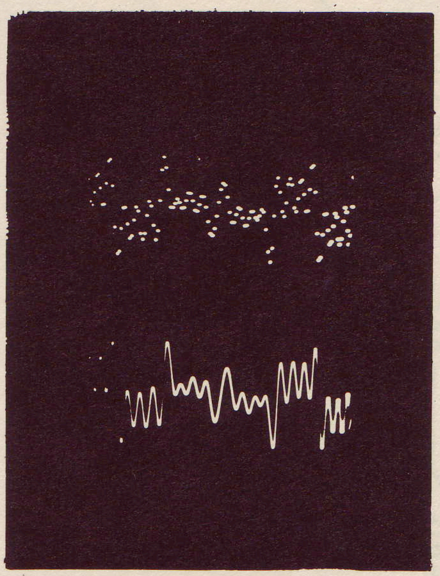
GRAPH NO. G65-25



UNFILTERED

FILTERED

$$f_{(x)} = 1/4\ [\sin 7x + \sin 8x + \sin 9x + \sin 10x]$$



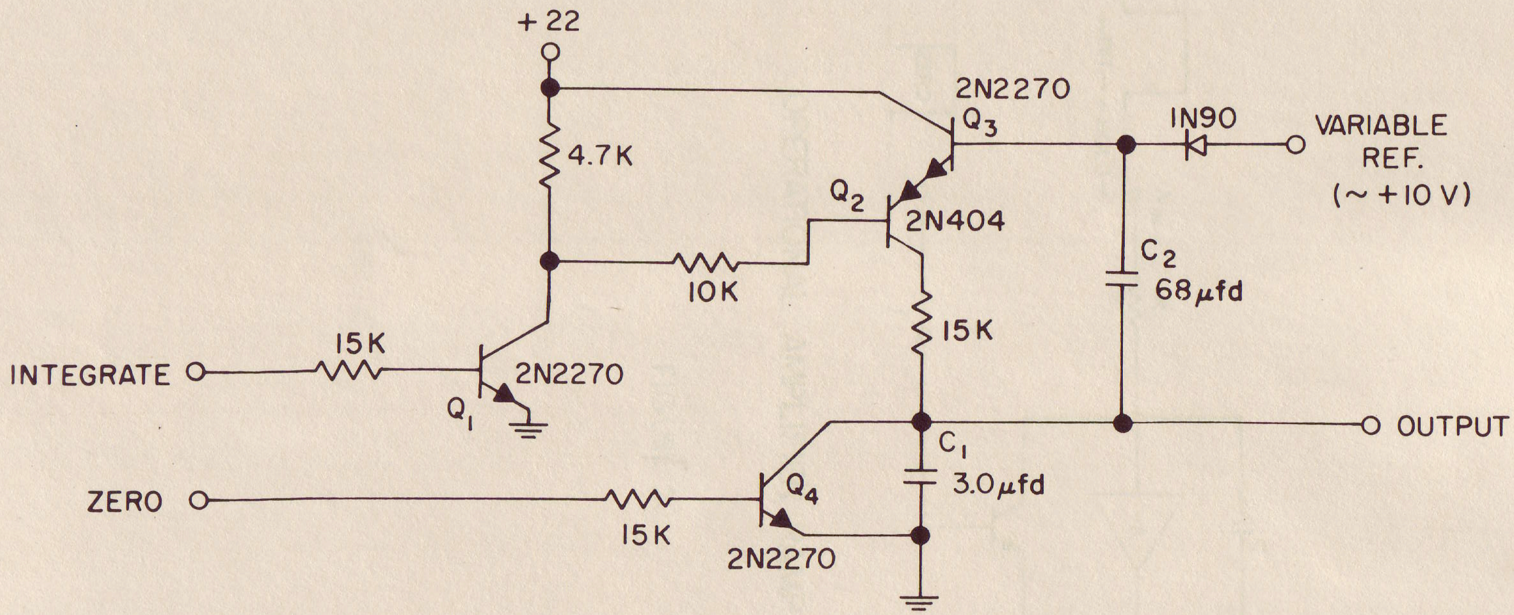
UNFILTERED

FILTERED

$$f_{(x)} = 1/6\ [\sin 2x + \sin 3x + \sin 7x + \sin 11x + \sin 15x + \sin 16x$$

COMPLEX SYNTHESIZED WAVEFORMS

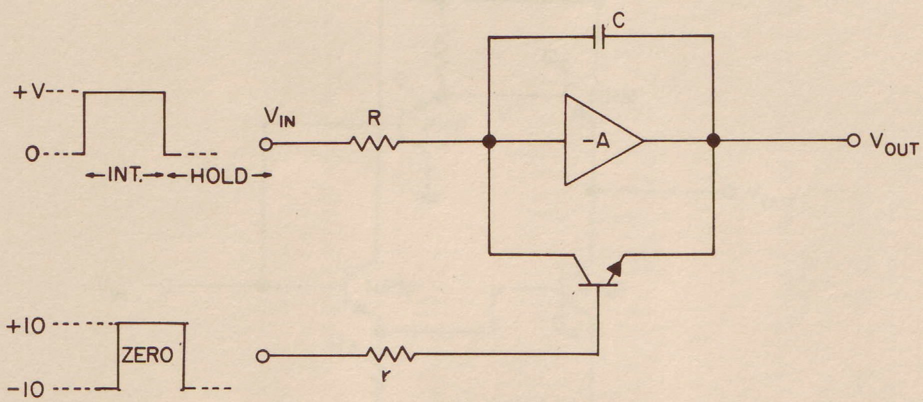
FIGURE 18



BOOT STRAP RAMP GENERATOR

FIGURE 19

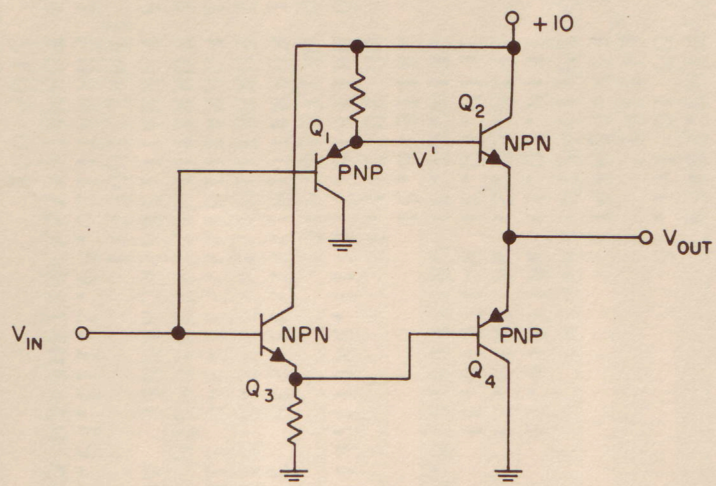
G65-119



OPERATIONAL AMPLIFIER RAMP GENERATOR

FIGURE 20

G65-120



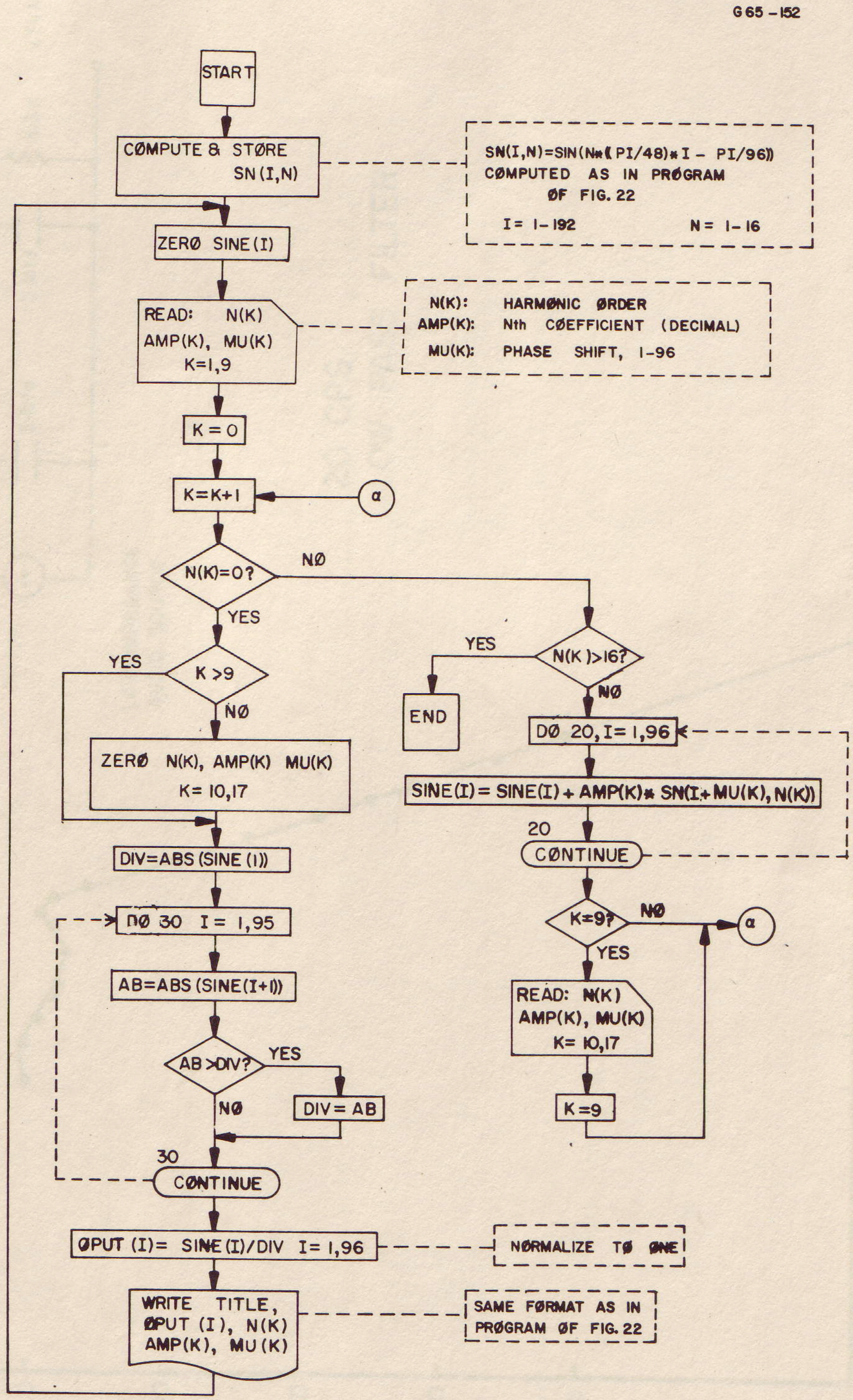
"ZERO-SHIFT" COMPLEMENTARY EMITTER-FOLLOWER

FIGURE 21

```
1720176 8 J CESSNA WAVFRM                       FORTRAN SOURCE LIST                    06/24/65
        ISN        SOURCE STATEMENT

          0 $IBFTC
          1        DIMENSION SN(96),L(96)
          2        PI=3.14159265
          3        PI1=PI/48.
          4        PI2=-PI/96.
          5        DO 30 N=1,16
          6        FN=N
          7        FSM=PI2
         10        FI=3.75*FN
         11        DO 10 I=1,96
         12        L(I)=I
         13        FSM=FSM+PI1
         14     10 SN(I)=SIN(FN*FSM)
         16        IF(N.EQ.1) WRITE(6,1)
         21        IF(N.EQ.2) WRITE(6,2)
         24        IF(N.EQ.3) WRITE(6,3)
         27        IF(N.GT.3) WRITE(6,4)N,N
         32        WRITE(6,5)
         33        DO 20 I=1,32
         34     20 WRITE(6,7)L(I),SN(I),L(I+32),SN(I+32),L(I+64),SN(I+64)
         36     30 WRITE(6,8)FI
         40      1 FORMAT(1H1////33X23H  SIN X  =  FUNDAMENTAL////)
         41      2 FORMAT(1H1////33X24H SIN 2X  =  2ND HARMONIC////)
         42      3 FORMAT(1H1////33X24H SIN 3X  =  3RD HARMONIC////)
         43      4 FORMAT(1H1////33X4HSIN I2,6HX  =  I2,11HTH HARMONIC////)
         44      5 FORMAT(15X62HCHANNEL  AMPLITUDE    CHANNEL  AMPLITUDE    CHANNEL
                  1AMPLITUDE//)
         45      7 FORMAT(I20,F9.2,I13,F9.2,I13,F9.2)
         46      8 FORMAT(////32X15HEACH CHANNEL = F6.2,8H DEGREES)
         47        STOP 001
         50        END
```

Figure 22



FLOW CHART FOURIER SERIES PROGRAM

FIGURE 23

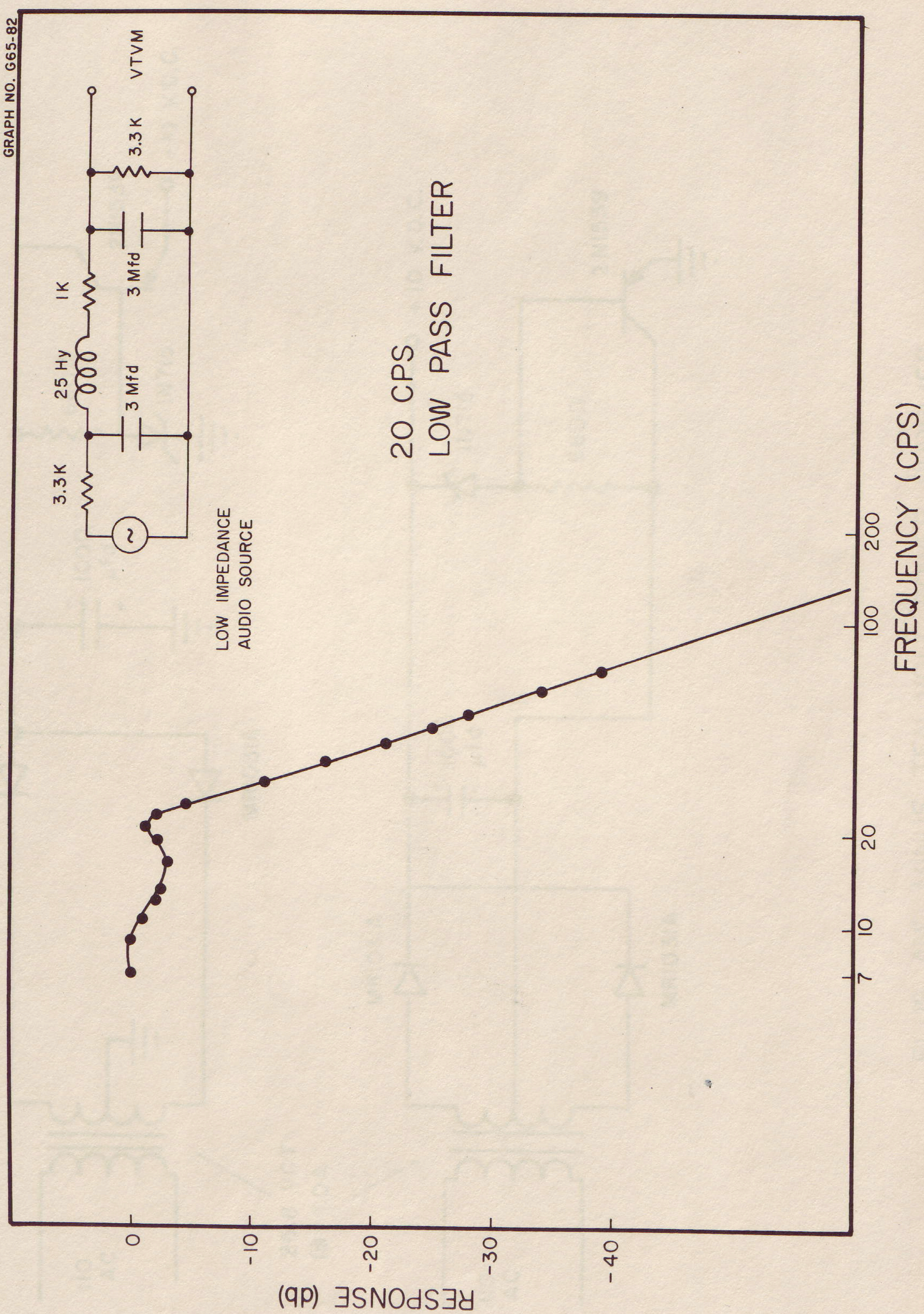
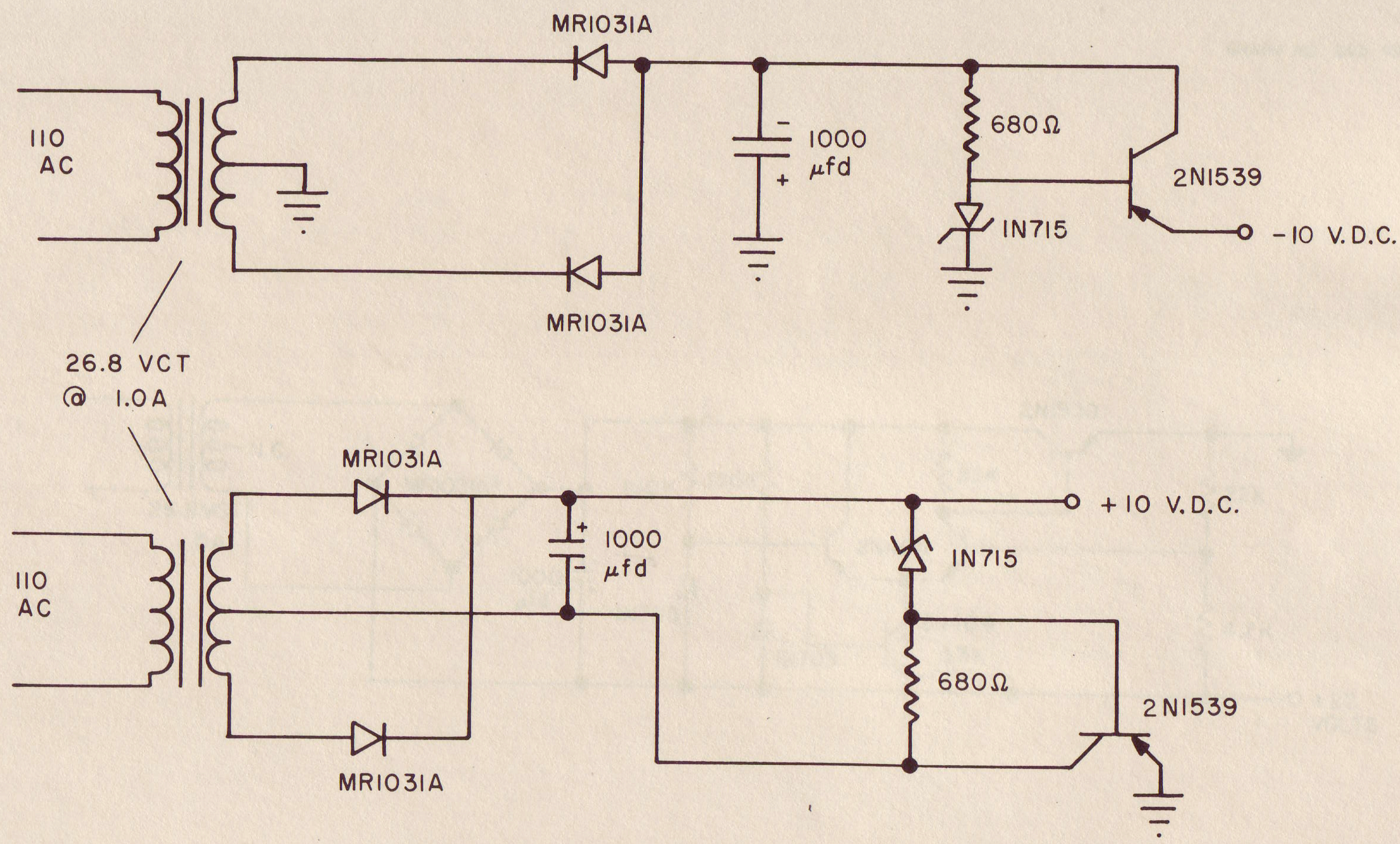

GRAPH NO. G65-82
3.3 K
25 Hy
1K
3 Mfd
3 Mfd
3.3 K
VTVM
LOW IMPEDANCE
AUDIO SOURCE
20 CPS
LOW PASS FILTER
0
-10
-20
-30
-40
RESPONSE (db)
7
10
20
100
200
FREQUENCY (CPS)

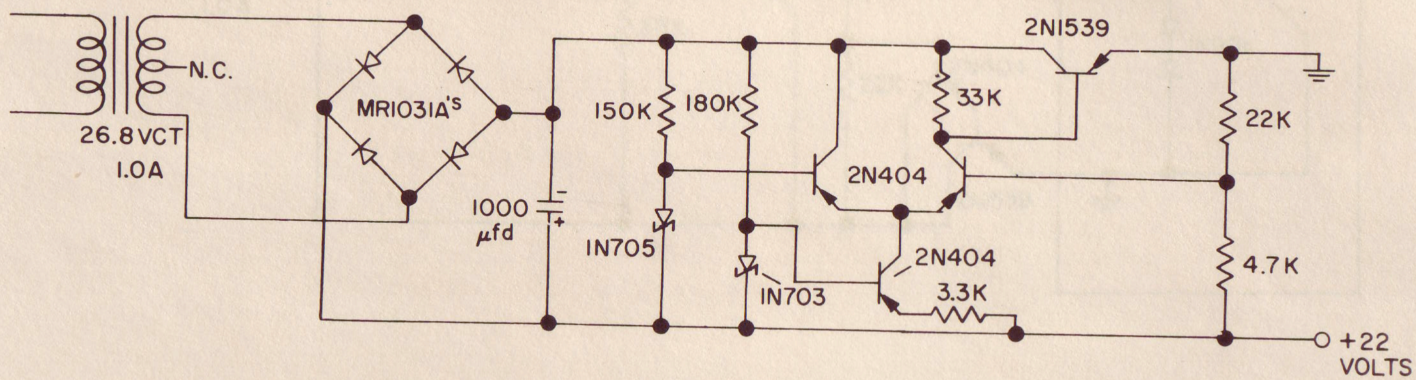
FIGURE 24



PLUS AND MINUS TEN VOLT POWER SUPPLIES

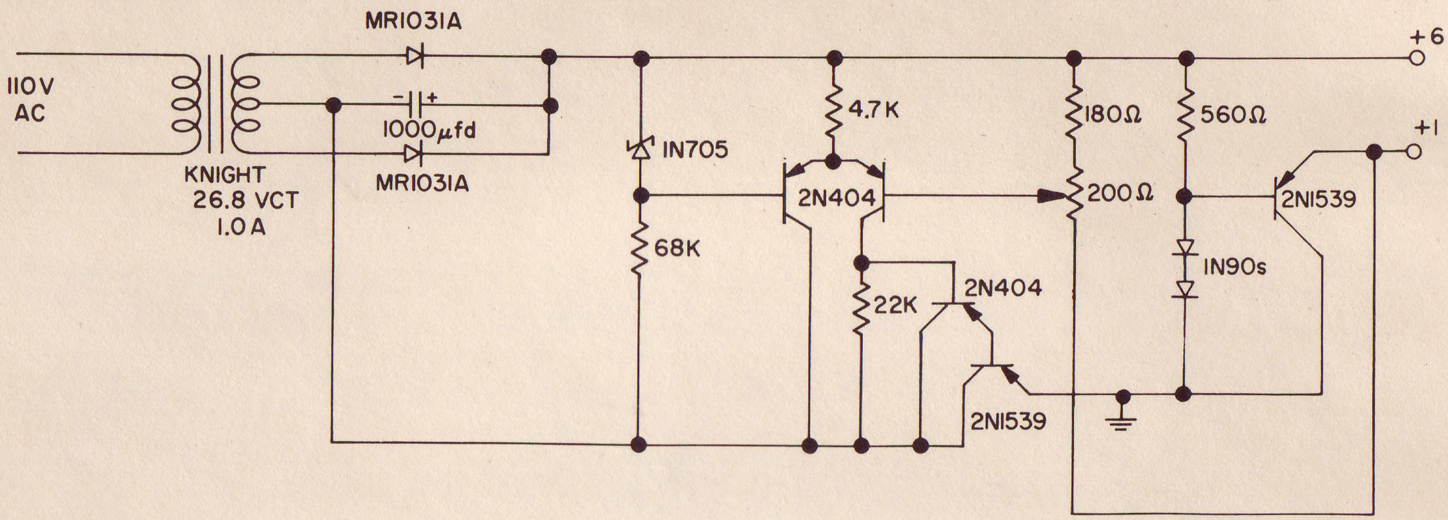
FIGURE 25

GRAPH NO. G65-49



22 VOLT POWER SUPPLY

FIGURE 26



POTENTIOMETER POWER SUPPLY

FIGURE 27